1 Given that

$$\frac{22}{(2x-3)(x+4)} \equiv \frac{A}{2x-3} + \frac{B}{x+4},$$

find the values of the constants A and B. **(3)**

2 Find the values of A, B and C such that

$$\frac{x+5}{(x+1)(x-3)^2} \equiv \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}.$$ **(4)**

3 Given that

$$\frac{4x^2-16x-7}{2x^2-9x+4} \equiv A + \frac{B}{2x-1} + \frac{C}{x-4},$$

find the values of the constants A, B and C. **(4)**

4 $$f(x) = 3x^3 + 11x^2 + 8x - 4.$$

a Fully factorise f(x). **(4)**

b Express $\dfrac{x+16}{f(x)}$ in partial fractions. **(4)**

5 Given that

$$f(x) = \frac{1}{x(2x-1)^2},$$

express f(x) in partial fractions. **(4)**

6 $$f(x) = \frac{x^3+5x^2-2x-19}{x^2+7x+10}.$$

Show that f(x) can be written in the form

$$f(x) = x + A + \frac{B}{x+2} + \frac{C}{x+5},$$

where A, B and C are integers to be found. **(5)**

7 The function f is defined by

$$f(x) = \frac{4}{x^2-1}.$$

a Express f(x) in partial fractions. **(3)**

The function g is defined by

$$g(x) = \frac{2+5x-x^2}{(x-4)(x-2)(x-1)}.$$

b Express g(x) in partial fractions. **(3)**

c Hence, or otherwise, solve the equation f(x) = g(x). **(5)**

1 **a** Expand $(1 - 4x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid. **(4)**

b By substituting $x = 0.01$ in your expansion, find the value of $\sqrt{6}$ to 6 significant figures. **(3)**

2 $$f(x) \equiv \frac{4}{1+2x-3x^2}.$$

a Express f(x) in partial fractions. **(3)**

b Hence, or otherwise, find the series expansion of f(x) in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid. **(5)**

3 **a** Expand $(2-x)^{-2}$, $|x| < 2$, in ascending powers of x up to and including the term in x^3. **(4)**

b Hence, find the coefficient of x^3 in the series expansion of $\dfrac{3-x}{(2-x)^2}$. **(2)**

4 $$f(x) \equiv \frac{4}{\sqrt{1+\frac{2}{3}x}}, \quad -\tfrac{3}{2} < x < \tfrac{3}{2}.$$

a Show that $f(\frac{1}{10}) = \sqrt{15}$. **(2)**

b Expand f(x) in ascending powers of x up to and including the term in x^2. **(3)**

c Use your expansion to obtain an approximation for $\sqrt{15}$, giving your answer as an exact, simplified fraction. **(2)**

d Show that $3\frac{55}{63}$ is a more accurate approximation for $\sqrt{15}$. **(2)**

5 **a** Expand $(1 - x)^{\frac{1}{3}}$, $|x| < 1$, in ascending powers of x up to and including the term in x^2. **(3)**

b By substituting $x = 10^{-3}$ in your expansion, find the cube root of 37 correct to 9 significant figures. **(3)**

6 The series expansion of $(1 + 5x)^{\frac{3}{5}}$, in ascending powers of x up to and including the term in x^3, is

$$1 + 3x + px^2 + qx^3, \quad |x| < \tfrac{1}{5}.$$

a Find the values of the constants p and q. **(4)**

b Use the expansion with a suitable value of x to find an approximate value for $(1.1)^{\frac{3}{5}}$. **(2)**

c Obtain the value of $(1.1)^{\frac{3}{5}}$ from your calculator and hence find the percentage error in your answer to part **b**. **(2)**

7 **a** Find the values of A, B and C such that

$$\frac{8-6x^2}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}.$$ **(4)**

b Hence find the series expansion of $\dfrac{8-6x^2}{(1+x)(2+x)^2}$, $|x| < 1$, in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(7)**

8 **a** Expand $(1-2x)^{\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^2. **(3)**

b By substituting $x = 0.0008$ in your expansion, find the square root of 39 correct to 7 significant figures. **(4)**

9 **a** Find the series expansion of $(1+8x)^{\frac{1}{3}}$, $|x| < \frac{1}{8}$, in ascending powers of x up to and including the term in x^2, simplifying each term. **(3)**

b Find the exact fraction k such that

$$\sqrt[3]{5} = k\sqrt[3]{1.08}$$ **(2)**

c Hence, use your answer to part **a** together with a suitable value of x to obtain an estimate for $\sqrt[3]{5}$, giving your answer to 4 significant figures. **(3)**

10
$$f(x) \equiv \frac{6x}{x^2-4x+3}, \quad |x| < 1.$$

a Express f(x) in partial fractions. **(3)**

b Show that for small values of x,

$$f(x) \approx 2x + \tfrac{8}{3}x^2 + \tfrac{26}{9}x^3.$$ **(5)**

11 **a** Find the binomial expansion of $(4+x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2 and state the set of values of x for which the expansion is valid. **(4)**

b By substituting $x = \frac{1}{20}$ in your expansion, find an estimate for $\sqrt{5}$, giving your answer to 9 significant figures. **(3)**

c Obtain the value of $\sqrt{5}$ from your calculator and hence comment on the accuracy of the estimate found in part **b**. **(2)**

12 **a** Expand $(1+2x)^{-\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3. **(4)**

b Hence, show that for small values of x,

$$\frac{2-5x}{\sqrt{1+2x}} \approx 2 - 7x + 8x^2 - \tfrac{25}{2}x^3.$$ **(3)**

c Solve the equation

$$\frac{2-5x}{\sqrt{1+2x}} = \sqrt{3}.$$ **(3)**

d Use your answers to parts **b** and **c** to find an approximate value for $\sqrt{3}$. **(2)**

13 **a** Expand $(1+x)^{-1}$, $|x| < 1$, in ascending powers of x up to and including the term in x^3. **(2)**

b Hence, write down the first four terms in the expansion in ascending powers of x of $(1+bx)^{-1}$, where b is a constant, for $|bx| < 1$. **(1)**

Given that in the series expansion of

$$\frac{1+ax}{1+bx}, \quad |bx| < 1,$$

the coefficient of x is –4 and the coefficient of x^2 is 12,

c find the values of the constants a and b, **(5)**

d find the coefficient of x^3 in the expansion. **(2)**

1 A curve has the equation

$$3x^2 + xy - y^2 + 9 = 0.$$

Find an expression for $\frac{dy}{dx}$ in terms of x and y. **(5)**

2 A curve has parametric equations

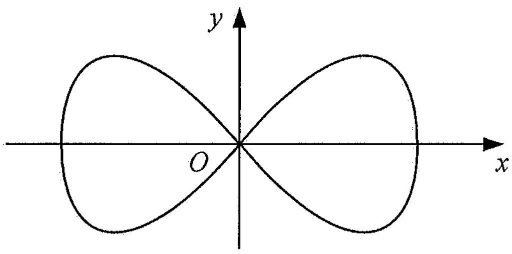
$$x = a\cos\theta,\quad y = a(\sin\theta - \theta),\quad 0 \le \theta < \pi,$$

where a is a positive constant.

a Show that $\frac{dy}{dx} = \tan\frac{\theta}{2}$. **(5)**

b Find, in terms of a, an equation for the tangent to the curve at the point where it crosses the y-axis. **(3)**

3



The diagram shows the curve with parametric equations

$$x = \cos\theta,\quad y = \tfrac{1}{2}\sin 2\theta,\quad 0 \le \theta < 2\pi.$$

a Find $\frac{dy}{dx}$ in terms of θ. **(3)**

b Find the two values of θ for which the curve passes through the origin. **(2)**

c Show that the two tangents to the curve at the origin are perpendicular to each other. **(2)**

d Find a cartesian equation for the curve. **(4)**

4 A curve has the equation

$$x^2 - 4xy + y^2 = 24.$$

a Show that $\frac{dy}{dx} = \frac{x - 2y}{2x - y}$. **(4)**

b Find an equation for the tangent to the curve at the point P (2, 10). **(3)**

The tangent to the curve at Q is parallel to the tangent at P.

c Find the coordinates of Q. **(4)**

5 A curve is given by the parametric equations

$$x = t^2 + 2,\quad y = t(t - 1).$$

a Find the coordinates of any points on the curve where the tangent to the curve is parallel to the x-axis. **(5)**

b Show that the tangent to the curve at the point (3, 2) has the equation

$$3x - 2y = 5.$$ **(5)**

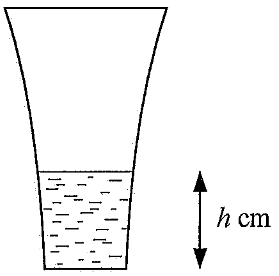
6 Find an equation for the normal to the curve with equation

$$x^3 - 3x + xy - 2y^2 + 3 = 0$$

at the point (1, 1).

Give your answer in the form $y = mx + c$. **(7)**

7



The diagram shows the cross-section of a vase. The volume of water in the vase, V cm^3, when the depth of water in the vase is h cm is given by

$$V = 40\pi(e^{0.1h} - 1).$$

The vase is initially empty and water is poured into it at a constant rate of 80 cm^3 s^{-1}.

Find the rate at which the depth of water in the vase is increasing

a when $h = 4$, **(5)**

b after 5 seconds of pouring water in. **(4)**

8 A curve is given by the parametric equations

$$x = \frac{t}{1+t}, \quad y = \frac{t}{1-t}, \quad t \neq \pm 1.$$

a Show that $\frac{dy}{dx} = \left(\frac{1+t}{1-t}\right)^2$. **(4)**

b Show that the normal to the curve at the point P, where $t = \frac{1}{2}$, has the equation

$$3x + 27y = 28.$$ **(4)**

The normal to the curve at P meets the curve again at the point Q.

c Find the exact value of the parameter t at Q. **(4)**

9 A curve has the equation

$$2x + x^2y - y^2 = 0.$$

Find the coordinates of the point on the curve where the tangent is parallel to the x-axis. **(8)**

10 A curve has parametric equations

$$x = a\sec\theta, \quad y = 2a\tan\theta, \quad -\frac{\pi}{2} \leq \theta < \frac{\pi}{2},$$

where a is a positive constant.

a Find $\frac{dy}{dx}$ in terms of θ. **(3)**

b Show that the normal to the curve at the point where $\theta = \frac{\pi}{4}$ has the equation

$$x + 2\sqrt{2}y = 5\sqrt{2}a.$$ **(4)**

c Find a cartesian equation for the curve in the form $y^2 = f(x)$. **(3)**

1 A curve has parametric equations

$$x = t^2, \quad y = \frac{2}{t}.$$

a Find $\frac{dy}{dx}$ in terms of t. **(3)**

b Find an equation for the normal to the curve at the point where $t = 2$, giving your answer in the form $y = mx + c$. **(3)**

2 A curve has the equation $y = 4^x$.

Show that the tangent to the curve at the point where $x = 1$ has the equation

$$y = 4 + 8(x - 1)\ln 2.$$ **(4)**

3 A curve has parametric equations

$$x = \sec\theta, \quad y = \cos 2\theta, \quad 0 \le \theta < \tfrac{\pi}{2}.$$

a Show that $\frac{dy}{dx} = -4\cos^3\theta$. **(4)**

b Show that the tangent to the curve at the point where $\theta = \frac{\pi}{6}$ has the equation

$$3\sqrt{3}\,x + 2y = k,$$

where k is an integer to be found. **(4)**

4 A curve has the equation

$$2x^2 + 6xy - y^2 + 77 = 0$$

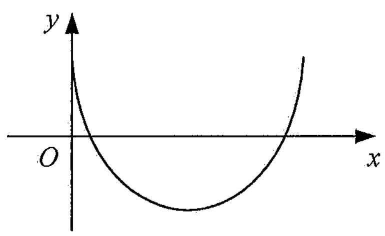
and passes through the point $P(2, -5)$.

a Show that the normal to the curve at P has the equation

$$x + y + 3 = 0.$$ **(6)**

b Find the x-coordinate of the point where the normal to the curve at P intersects the curve again. **(3)**

5



The diagram shows the curve with parametric equations

$$x = \theta - \sin\theta, \quad y = \cos\theta, \quad 0 \le \theta \le 2\pi.$$

a Find the exact coordinates of the points where the curve crosses the x-axis. **(3)**

b Show that $\frac{dy}{dx} = -\cot\frac{\theta}{2}$. **(5)**

c Find the exact coordinates of the point on the curve where the tangent to the curve is parallel to the x-axis. **(2)**

6 A curve has parametric equations

$$x = \sin\theta, \quad y = \sec^2\theta, \quad -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}.$$

The point P on the curve has x-coordinate $\tfrac{1}{2}$.

a Write down the value of the parameter θ at P. **(1)**

b Show that the tangent to the curve at P has the equation

$$16x - 9y + 4 = 0.$$ **(6)**

c Find a cartesian equation for the curve. **(2)**

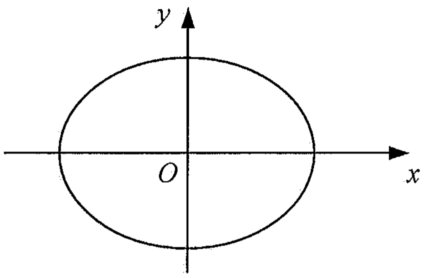
7 A curve has the equation

$$2\sin x - \tan 2y = 0.$$

a Show that $\dfrac{dy}{dx} = \cos x \cos^2 2y$. **(4)**

b Find an equation for the tangent to the curve at the point $(\tfrac{\pi}{3}, \tfrac{\pi}{6})$, giving your answer in the form $ax + by + c = 0$. **(3)**

8



A particle moves on the ellipse shown in the diagram such that at time t its coordinates are given by

$$x = 4\cos t, \quad y = 3\sin t, \quad t \geq 0.$$

a Find $\dfrac{dy}{dx}$ in terms of t. **(3)**

b Show that at time t, the tangent to the path of the particle has the equation

$$3x\cos t + 4y\sin t = 12.$$ **(3)**

c Find a cartesian equation for the path of the particle. **(3)**

9 The curve with parametric equations

$$x = \frac{t}{t+1}, \quad y = \frac{2t}{t-1},$$

passes through the origin, O.

a Show that $\dfrac{dy}{dx} = -2\left(\dfrac{t+1}{t-1}\right)^2$. **(4)**

b Find an equation for the normal to the curve at O. **(2)**

c Find the coordinates of the point where the normal to the curve at O meets the curve again. **(4)**

d Show that the cartesian equation of the curve can be written in the form

$$y = \frac{2x}{2x-1}.$$ **(4)**

C4 INTEGRATION Worksheet N

1 Show that

$$\int_2^7 \frac{8}{4x-3}\,dx = \ln 25. \qquad \textbf{(4)}$$

2 Given that $y = \frac{\pi}{4}$ when $x = 1$, solve the differential equation

$$\frac{dy}{dx} = x \sec y \operatorname{cosec}^3 y. \qquad \textbf{(7)}$$

3 **a** Use the trapezium rule with three intervals of equal width to find an approximate value for the integral

$$\int_0^{1.5} e^{x^2-1}\,dx. \qquad \textbf{(4)}$$

b Use the trapezium rule with six intervals of equal width to find an improved approximation for the above integral. **(2)**

4

$$f(x) \equiv \frac{3(2-x)}{(1-2x)^2(1+x)}.$$

a Express f(x) in partial fractions. **(4)**

b Show that

$$\int_1^2 f(x)\,dx = 1 - \ln 2. \qquad \textbf{(6)}$$

5 The rate of growth in the number of yeast cells, N, present in a culture after t hours is proportional to N.

a By forming and solving a differential equation, show that

$$N = Ae^{kt},$$

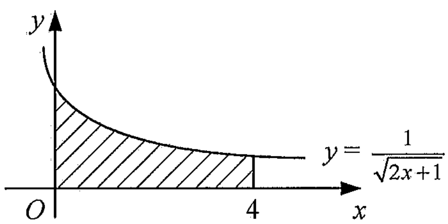
where A and k are positive constants. **(4)**

Initially there are 200 yeast cells in the culture and after 2 hours there are 3000 yeast cells in the culture. Find, to the nearest minute, after how long

b there are 10 000 yeast cells in the culture, **(5)**

c the number of yeast cells is increasing at the rate of 5 per second. **(4)**

6



The diagram shows part of the curve with equation $y = \frac{1}{\sqrt{2x+1}}$.

The shaded region is bounded by the curve, the coordinate axes and the line $x = 4$.

a Find the area of the shaded region. **(4)**

The shaded region is rotated through four right angles about the x-axis.

b Find the volume of the solid formed, giving your answer in the form $\pi \ln k$. **(5)**

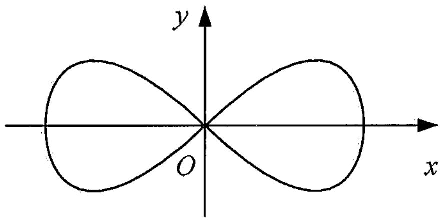
7 Using the substitution $u^2 = x + 3$, show that

$$\int_0^1 x\sqrt{x+3}\ \mathrm{d}x = k(3\sqrt{3} - 4),$$

where k is a rational number to be found. **(7)**

8 **a** Use the identities for $\sin(A + B)$ and $\sin(A - B)$ to prove that

$$2\sin A\cos B \equiv \sin(A + B) + \sin(A - B).$$ **(2)**



The diagram shows the curve given by the parametric equations

$$x = 2\sin 2t,\quad y = \sin 4t,\quad 0 \le t < \pi.$$

b Show that the total area enclosed by the two loops of the curve is given by

$$\int_0^{\frac{\pi}{4}} 16\sin 4t\cos 2t\ \mathrm{d}t.$$ **(4)**

c Evaluate this integral. **(5)**

9 $$\mathrm{f}(x) \equiv \frac{x^2 - 22}{(x+2)(x-4)}.$$

a Find the values of the constants A, B and C such that

$$\mathrm{f}(x) \equiv A + \frac{B}{x+2} + \frac{C}{x-4}.$$ **(3)**

The finite region R is bounded by the curve $y = \mathrm{f}(x)$, the coordinate axes and the line $x = 2$.

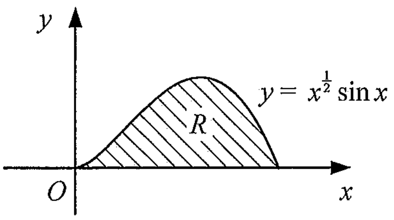
b Find the area of R, giving your answer in the form $p + \ln q$, where p and q are integers. **(5)**

10 **a** Find $\int \sin^2 x\ \mathrm{d}x$. **(4)**

b Use integration by parts to show that

$$\int x\sin^2 x\ \mathrm{d}x = \tfrac{1}{8}(2x^2 - 2x\sin 2x - \cos 2x) + c,$$

where c is an arbitrary constant. **(4)**



The diagram shows the curve with equation $y = x^{\frac{1}{2}}\sin x$, $0 \le x \le \pi$.

The finite region R, bounded by the curve and the x-axis, is rotated through 2π radians about the x-axis.

c Find the volume of the solid formed, giving your answer in terms of π. **(3)**

1 **a** Express $\dfrac{1}{x^2 - 3x + 2}$ in partial fractions. **(3)**

b Show that

$$\int_3^4 \frac{1}{x^2 - 3x + 2}\, \mathrm{d}x = \ln \frac{a}{b},$$

where a and b are integers to be found. **(5)**

2 Evaluate

$$\int_0^{\frac{\pi}{6}} \cos x \cos 3x \, \mathrm{d}x.$$ **(6)**

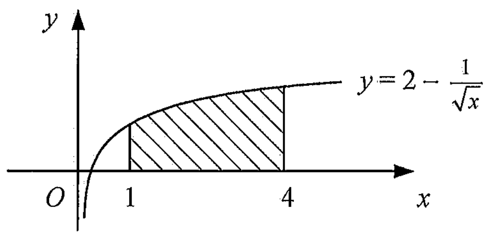
3 **a** Find the quotient and remainder obtained in dividing $(x^2 + x - 1)$ by $(x - 1)$. **(3)**

b Hence, show that

$$\int \frac{x^2 + x - 1}{x - 1}\, \mathrm{d}x = \tfrac{1}{2}x^2 + 2x + \ln|x - 1| + c,$$

where c is an arbitrary constant. **(2)**

4



The diagram shows the curve with equation $y = 2 - \dfrac{1}{\sqrt{x}}$.

The shaded region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through $360°$ about the x-axis to form the solid S.

a Show that the volume of S is $2\pi(2 + \ln 2)$. **(6)**

S is used to model the shape of a container with 1 unit on each axis representing 10 cm.

b Find the volume of the container correct to 3 significant figures. **(2)**

5 **a** Use integration by parts to find $\int x \ln x \, \mathrm{d}x$. **(4)**

b Given that $y = 4$ when $x = 2$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = xy \ln x, \quad x > 0, \quad y > 0,$$

and hence, find the exact value of y when $x = 1$. **(5)**

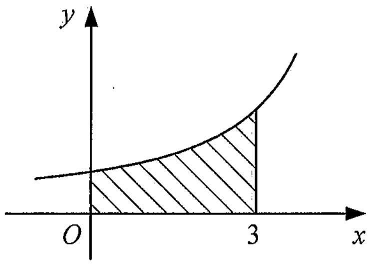
6 **a** Evaluate $\int_0^{\frac{\pi}{3}} \sin x \sec^2 x \, \mathrm{d}x$. **(4)**

b Using the substitution $u = \cos\theta$, or otherwise, show that

$$\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos^4\theta}\, \mathrm{d}\theta = a + b\sqrt{2},$$

where a and b are rational. **(6)**

7



The diagram shows part of the curve with parametric equations

$$x = 2t + 1, \quad y = \frac{1}{2-t}, \quad t \neq 2.$$

The shaded region is bounded by the curve, the coordinate axes and the line $x = 3$.

a Find the value of the parameter t at the points where $x = 0$ and where $x = 3$. **(2)**

b Show that the area of the shaded region is $2 \ln \frac{5}{2}$. **(5)**

c Find the exact volume of the solid formed when the shaded region is rotated completely about the x-axis. **(5)**

8 **a** Using integration by parts, find

$$\int 6x \cos 3x \ \mathrm{d}x.$$ **(5)**

b Use the substitution $x = 2 \sin u$ to show that

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} \ \mathrm{d}x = \frac{\pi}{3}.$$ **(5)**

9 In an experiment to investigate the formation of ice on a body of water, a thin circular disc of ice is placed on the surface of a tank of water and the surrounding air temperature is kept constant at –5°C.

In a model of the situation, it is assumed that the disc of ice remains circular and that its area, A cm^2 after t minutes, increases at a rate proportional to its perimeter.

a Show that

$$\frac{\mathrm{d}A}{\mathrm{d}t} = k\sqrt{A},$$

where k is a positive constant. **(3)**

b Show that the general solution of this differential equation is

$$A = (pt + q)^2,$$

where p and q are constants. **(4)**

Given that when $t = 0$, $A = 25$ and that when $t = 20$, $A = 40$,

c find how long it takes for the area to increase to 50 cm^2. **(5)**

10

$$\mathrm{f}(x) \equiv \frac{5x+1}{(1-x)(1+2x)}.$$

a Express f(x) in partial fractions. **(3)**

b Find $\int_0^{\frac{1}{2}} \mathrm{f}(x) \ \mathrm{d}x$, giving your answer in the form $k \ln 2$. **(4)**

c Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3, for $|x| < \frac{1}{2}$. **(6)**

C4 VECTORS Worksheet E

1 Relative to a fixed origin, the line l has vector equation

$$\mathbf{r} = \mathbf{i} - 4\mathbf{j} + p\mathbf{k} + \lambda(2\mathbf{i} + q\mathbf{j} - 3\mathbf{k}),$$

where λ is a scalar parameter.

Given that l passes through the point with position vector $(7\mathbf{i} - \mathbf{j} - \mathbf{k})$,

a find the values of the constants p and q, **(3)**

b find, in degrees, the acute angle l makes with the line with equation

$$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(-4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}).$$ **(4)**

2 The points A and B have position vectors $\begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 0 \\ -6 \end{pmatrix}$ respectively, relative to a fixed origin.

a Find, in vector form, an equation of the line l which passes through A and B. **(2)**

The line m has equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ -5 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}.$$

Given that lines l and m intersect at the point C,

b find the position vector of C, **(5)**

c show that C is the mid-point of AB. **(2)**

3 Relative to a fixed origin, the points P and Q have position vectors $(5\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and $(3\mathbf{i} + \mathbf{j})$ respectively.

a Find, in vector form, an equation of the line L_1 which passes through P and Q. **(2)**

The line L_2 has equation

$$\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} - \mathbf{k} + \mu(5\mathbf{i} - \mathbf{j} + 3\mathbf{k}).$$

b Show that lines L_1 and L_2 intersect and find the position vector of their point of intersection. **(6)**

c Find, in degrees to 1 decimal place, the acute angle between lines L_1 and L_2. **(4)**

4 Relative to a fixed origin, the lines l_1 and l_2 have vector equations as follows:

$$l_1: \quad \mathbf{r} = 5\mathbf{i} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}),$$

$$l_2: \quad \mathbf{r} = 7\mathbf{i} - 3\mathbf{j} + 7\mathbf{k} + \mu(-\mathbf{i} + \mathbf{j} - 2\mathbf{k}),$$

where λ and μ are scalar parameters.

a Show that lines l_1 and l_2 intersect and find the position vector of their point of intersection. **(6)**

The points A and C lie on l_1 and the points B and D lie on l_2.

Given that $ABCD$ is a parallelogram and that A has position vector $(9\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$,

b find the position vector of C. **(3)**

Given also that the area of parallelogram $ABCD$ is 54,

c find the distance of the point B from the line l_1. **(4)**

5 Relative to a fixed origin, the points A and B have position vectors $(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$ and $(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ respectively.

a Find, in vector form, an equation of the line l_1 which passes through A and B. **(2)**

The line l_2 passes through the point C with position vector $(4\mathbf{i} - 7\mathbf{j} - \mathbf{k})$ and is parallel to the vector $(6\mathbf{j} - 2\mathbf{k})$.

b Write down, in vector form, an equation of the line l_2. **(1)**

c Show that A lies on l_2. **(2)**

d Find, in degrees, the acute angle between lines l_1 and l_2. **(4)**

6 The points A and B have position vectors $\begin{pmatrix} 5 \\ -1 \\ -10 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 1 \\ -8 \end{pmatrix}$ respectively, relative to a fixed origin O.

a Find, in vector form, an equation of the line l which passes through A and B. **(2)**

The line l intersects the y-axis at the point C.

b Find the coordinates of C. **(2)**

The point D on the line l is such that OD is perpendicular to l.

c Find the coordinates of D. **(5)**

d Find the area of triangle OCD, giving your answer in the form $k\sqrt{5}$. **(3)**

7 Relative to a fixed origin, the line l_1 has the equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -6 \\ -2 \end{pmatrix} + s\begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}.$$

a Show that the point P with coordinates $(1, 6, -5)$ lies on l_1. **(1)**

The line l_2 has the equation

$$\mathbf{r} = \begin{pmatrix} 4 \\ -4 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix},$$

and intersects l_1 at the point Q.

b Find the position vector of Q. **(3)**

The point R lies on l_2 such that $PQ = QR$.

c Find the two possible position vectors of the point R. **(5)**

8 Relative to a fixed origin, the points A and B have position vectors $(4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k})$ and $(4\mathbf{i} + 6\mathbf{j} + 2\mathbf{k})$ respectively.

a Find, in vector form, an equation of the line l_1 which passes through A and B. **(2)**

The line l_2 has equation

$$\mathbf{r} = \mathbf{i} + 5\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

b Show that l_1 and l_2 intersect and find the position vector of their point of intersection. **(4)**

c Find the acute angle between lines l_1 and l_2. **(3)**

d Show that the point on l_2 closest to A has position vector $(-\mathbf{i} + 3\mathbf{j} - \mathbf{k})$. **(5)**

1 The points A and B have position vectors $\begin{pmatrix} 2 \\ -1 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix}$ respectively, relative to a fixed origin.

a Find, in vector form, an equation of the line l which passes through A and B. **(2)**

The line m has equation

$$\mathbf{r} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} a \\ -3 \\ 1 \end{pmatrix},$$

where a is a constant.

Given that lines l and m intersect,

b find the value of a and the coordinates of the point where l and m intersect. **(6)**

2 Relative to a fixed origin, the points A, B and C have position vectors $(-4\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, $(2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k})$ and $(6\mathbf{i} + 4\mathbf{j} + \mathbf{k})$ respectively.

a Show that $\cos(\angle ABC) = \frac{1}{3}$. **(3)**

The point M is the mid-point of AC.

b Find the position vector of M. **(2)**

c Show that BM is perpendicular to AC. **(3)**

d Find the size of angle ACB in degrees. **(3)**

3 Relative to a fixed origin O, the points A and B have position vectors $\begin{pmatrix} 9 \\ 5 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 11 \\ 7 \\ -3 \end{pmatrix}$ respectively.

a Find, in vector form, an equation of the line L which passes through A and B. **(2)**

The point C lies on L such that OC is perpendicular to L.

b Find the position vector of C. **(5)**

c Find, to 3 significant figures, the area of triangle OAC. **(3)**

d Find the exact ratio of the area of triangle OAB to the area of triangle OAC. **(2)**

4 Relative to a fixed origin O, the points A and B have position vectors $(7\mathbf{i} - 5\mathbf{j} - \mathbf{k})$ and $(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ respectively.

a Find $\cos(\angle AOB)$, giving your answer in the form $k\sqrt{6}$, where k is an exact fraction. **(4)**

b Show that AB is perpendicular to OB. **(3)**

The point C is such that $\overrightarrow{OC} = \frac{3}{2}\overrightarrow{OB}$.

c Show that AC is perpendicular to OA. **(3)**

d Find the size of $\angle ACO$ in degrees to 1 decimal place. **(3)**