CMI/PROMYS International Alliance

PROMYS, June 30 to August 10, 2013

Oxford Masterclasses, August 11 to August 17, 2013

In the summer of 2013, the International Alliance of the Program in Mathematics for Young Scientists (PROMYS) and the Clay Mathematics Institute (CMI) will provide merit-based scholarships to cover tuition, airfare, plus room and board for 10 European students to engage in PROMYS's intensive 6-week immersion program at Boston University with a one-week follow-up in the Combinatorics Masterclasses at Oxford University. Secondary students resident in Europe, including graduating seniors, may apply. Participants must be at least 16 years old by August 11, 2013. Applicants should complete the application (below), which includes a problem set, secondary school transcript, and teacher recommendation.

We realize that some applicants may not have regularly scheduled summer vacation at the same time the PROMYS and Oxford Masterclasses are in session. We will be happy to speak with the schools of accepted students to assist in the resolution of any conflicts with school attendance dates. Visit http://www.promys.org for more information.

Program Structure. An ambitious group of approximately 80 high school students will gather at Boston University, in Massachusetts, USA to engage in intensive study of a significant piece of mathematics. They will work with members of our faculty and be assisted by experienced undergraduates who are embarking on their own mathematical careers at some of the finest universities.

Each morning, participants will attend a Number Theory class; more experienced students may also attend an advanced seminar. They will then work independently and in small groups on problem sets distributed at the end of each class meeting. The problems will encourage students to design their own numerical experiments and employ their own powers of observation, as they discover mathematical patterns, formulate and test conjectures, and justify their ideas by devising their own mathematical proofs. In addition, each student will be supervised by a counselor in residence who will always be available for helpful discussions.

After the close of program in Boston, there will be further Combinatorics Masterclasses at Oxford University, directed by Professor David Conlon. These will be similar in spirit to the activities held at PROMYS, looking closely at an advanced topic in Combinatorics, and will be accompanied by a series of guest lectures.

Students who find their PROMYS experience especially worthwhile may apply to return for further studies in the following summer. In 2013, PROMYS will offer advanced seminars in Geometry and Symmetry, Wavelet Transformations, and Representation Theory. Advanced participants also work on independent research projects in mathematics. Research projects of past summers have ranged from combinatorics, tetrahedral packings of 3-space, Ramanujan's q-calculus, continued fractions and geodesics in the Poincaré upper half-plane, quaternion algebras and quadratic forms, to name only a few. Counselors and returning students will also organize their own seminars on topics of interest to members of the program.

Program Goals. Mathematics may well be the most widely misunderstood branch of the sciences. Young people contemplating careers in science find it difficult to imagine what a research mathematician really does. One common image features a mathematician programming a computer to do difficult calculations. Another pictures a lone mathematician working in isolation on ideas so abstruse that no normal person could comprehend them. Neither of these images comes close to capturing the spirit of mathematical inquiry. It is certainly true that many modern mathematicians use computers to perform numerical and geometrical experiments. But this experimental phase is only one component of the mathematical experience, and the use of computers in this phase is the exception, not the rule. Nor is it true that mathematicians work in isolation. Indeed, a distinctive feature of mathematics is the open sharing of ideas within a community nurtured by a common language, shared values, and shared goals.

All too often, mathematics is presented to students as a highly polished and well organized collection of definitions, algorithms, and theorems. The long struggle of many individuals that culminated in this "finished product" remains a hidden secret. Students rarely learn of the dynamic nature of mathematics, nor do they see the creative side. They do not come to understand that mathematics is a thriving field of research activity which is progressing faster today than at any other time in its distinguished history.

These misunderstandings may stem from the fact that mathematics deals so heavily in ideas. In mathematics, perhaps more than in any other science, research is an activity of the mind. The primary goal of the mathematician is to *understand* – to discover the essential ingredients of complex systems in order to render them simple, to find order within apparent chaos, to draw analogies between different structures, and to find connections between seemingly disparate branches of mathematics and science. To make interesting new contributions in the field of mathematics requires a healthy mix of creativity, experience, and hard work.

We aim to engage young people in the struggle to understand an intricate collection of significant mathematical ideas. Participants come with unbounded energy and are anxious to grapple with challenging ideas. At the beginning of their investigations, they may sometimes feel lost and perplexed. But through carefully designed problem sets, we hope to subtly direct students along productive paths towards understanding—to suggest that they experiment with examples and formulate conjectures, to encourage them to ask good questions, and to help them realize that through careful thought they can penetrate formidable obstacles and invent their own answers to difficult questions. The attitudes acquired through this experience will be far more valuable than the particular topics mastered.

Professor Glenn Stevens
Director of PROMYS

PROMYS gratefully acknowledges the financial support of its sponsors:

Boston University
Clay Mathematics Institute/PROMYS partnership
American Mathematical Society
The Linde Family Foundation
National Security Agency
The PROMYS Foundation

Please visit our website http://www.promys.org for links to each of these institutions.

Application Form for
CMI/PROMYS International Alliance
June 30 to August 17, 2013

Please complete this form and return it along with an official transcript from your school to: PROMYS, Department of Mathematics and Statistics, Boston University, 111 Cummington Mall, Room 142, Boston, Massachusetts 02215.

Application deadline is May 1, 2013. Finalists will be contacted by email to arrange an interview. Interviews may, in some cases, be by Skype or telephone. Admission decisions will be made on a rolling basis beginning April 1.

Name: ____

Address: ____

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Email: ________________ Date of Birth: ________________ Gender: Male ___ Female ___

Parent/Guardian Name: ____

Email: ______________________ Telephone: ______________________

Name of Secondary School: ____

Secondary School Address: ____

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Projected Graduation Date (month/year): ______________________________

Please ask one of your mathematics teachers to write a letter of recommendation on the enclosed form.

Teacher's Name: ______________________________ Email: ____________________

Please tell us about yourself by answering the following questions. If you need more space, please attach another sheet of paper.

a. Have you ever participated in a special program in mathematics or science before? If so, tell us briefly about that experience.

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b. Tell us about any other mathematical experiences you have had. For example, have you ever done a special mathematics project, or entered a mathematics competition?

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c. What other interests and hobbies do you have?

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d. What do you hope to gain by coming to PROMYS and attending the Oxford Masterclasses?

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Do you wish to identify yourself as belonging to any racial or ethnic groups? Yes ___ No ___

If so, to which groups do you belong? ______________________________________

Please tell us how you learned of the PROMYS program. _________________________

Applicant's signature: _______________________ Date: ____________________

The Problems

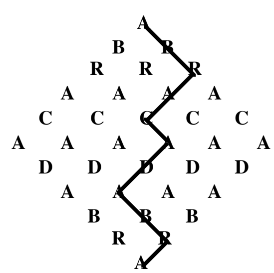
Please attempt each of the following problems. Though they can all be solved with no more than a standard high school mathematics background, most of the problems require considerably more ingenuity than is usually expected in high school. You should keep in mind that we do not expect you to find complete solutions to all of them. Rather, we are looking to see how you approach challenging problems. Here are a few suggestions:

- Think carefully about the meaning of each problem.
- Examine special cases, either through numerical examples or by drawing pictures.
- Be bold in making conjectures.
- Test your conjectures through further experimentation, and try to devise mathematical proofs to support the surviving ones.
- Can you solve special cases of a problem, or state and solve simpler but related problems?

If you think you know the answer to a question, but cannot prove that your answer is correct, tell us what kind of evidence you have found to support your belief. If you use books or articles in your explorations, be sure to cite your sources.

You may find that most of the problems require some patience. Do not rush through them. It is not unreasonable to spend a month or more thinking about the problems. It might be good strategy to devote most of your time to a small selection of problems which you find especially interesting. Be sure to tell us about progress you have made on problems not yet completely solved. **For each problem you solve, please justify your answer clearly and tell us how you arrived at your solution.**

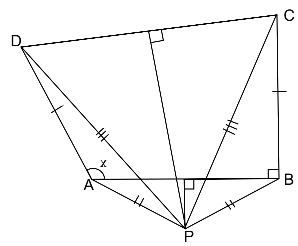
1. How many ways are there of spelling out "ABRACADABRA" by traversing the following diamond, always going from one letter to an adjacent one? One of the ways is shown.



2. Consider the sequence $t_0 = 3,\ t_1 = 3^3,\ t_2 = 3^{3^3},\ t_3 = 3^{3^{3^3}},\ \ldots$ defined by $t_0 = 3$ and $t_{n+1} = 3^{t_n}$ for $n \geq 0$. What are the last two digits in $t_3 = 3^{3^{3^3}}$? Can you say what the last *three* digits are? Show that the last 10 digits of t_k are the same for all $k \geq 10$.

3. In the game of Martian basketball, a points are given for a free throw and b points are given for a field goal, where a and b are positive integers. If $a = 2$ and $b = 5$, then it is not possible for a team to score exactly 1 point. Nor is it possible to score exactly 3 points. Are there any other unattainable scores? How many unattainable scores are there if $a = 3$ and $b = 5$? Is it true for any choice of a and b that there are only finitely many unattainable scores? Suppose a and b are unknown, but it is known that neither a nor b is equal to 2 and that there are exactly 65 unattainable scores. Can you determine a and b? Explain.

4. According to the Journal of Irreproducible Results, any obtuse angle is a right angle! Here is their argument.



Given the obtuse angle x, we make a quadrilateral $ABCD$ with $\angle DAB = x$, and $\angle ABC = 90°$, and $AD = BC$. Say the perpendicular bisector to DC meets the perpendicular bisector to AB at P. Then $PA = PB$ and $PC = PD$. So the triangles PAD and PBC have equal sides and are congruent. Thus $\angle PAD = \angle PBC$. But PAB is isosceles, hence $\angle PAB = \angle PBA$. Subtracting, gives $x = \angle PAD - \angle PAB = \angle PBC - \angle PBA = 90°$. This is a preposterous conclusion – just where is the mistake in the "proof" and why does the argument break down there?

5. Show that there are no positive integers n for which $n^4 + 2n^3 + 2n^2 + 2n + 1$ is a perfect square. Are there any positive integers n for which $n^4 + n^3 + n^2 + n + 1$ is a perfect square? If so, find all such n.

6. The squares of an infinite chessboard are numbered as follows: in the zeroth row and column we put 0, and then in every other square we put the smallest non-negative integer that does not appear anywhere below it in the same column nor anywhere to the left of it in the same row.

...	...						
6	7	...					
5	4	7	...				
4	5	6	7	...			
3	2	1	0	7	...		
2	3	0	1	6	7	...	
1	0	3	2	5	4	7	...
0	1	2	3	4	5	6	...

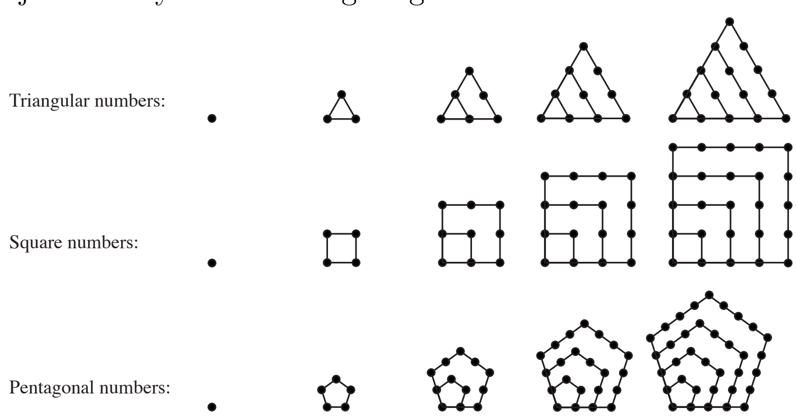
What number will appear in the 2013th row and 1989th column? Can you generalize?

7. Find a positive integer m such that $\frac{1}{2}m$ is a perfect square and $\frac{1}{3}m$ is a perfect cube. Can you find a positive integer n for which $\frac{1}{2}n$ is a perfect square, $\frac{1}{3}n$ is a perfect cube and $\frac{1}{5}n$ is a perfect fifth power?

8. Let's agree to say that a positive integer is *prime-like* if it is not divisible by 2, 3, or 5. How many prime-like positive integers are there less than 100? less than 1000? A positive integer is *very prime-like* if it is not divisible by any prime less than 15. How many very prime-like positive integers are there less than 90000? Without giving an exact answer, can you say *approximately* how many very prime-like positive integers are less than 10^{10}? less than 10^{100}? Explain your reasoning as carefully as you can.

9. Let P_0 be an equilateral triangle of area 1. Each side of P_0 is trisected, and the corners are snipped off, creating a new polygon (in fact, a hexagon) P_1. What is the area of P_1? Now repeat the process to P_1 – i.e. trisect each side and snip off the corners – to obtain a new polygon P_2. What is the area of P_2? Now repeat this process infinitely often to create an object P_∞. What is the area of P_∞?

10. The triangular numbers are the numbers $1, 3, 6, 10, 15, \ldots$; the square numbers are the numbers $1, 4, 9, 16, 25, \ldots$ The pentagonal numbers $1, 5, 12, 22, 35, \ldots$. The geometrical language is justified by the following diagrams:



a. What are the first five hexagonal numbers? What are the first five septagonal numbers? What are the first five r-gonal numbers? Give a formula for the nth triangular number. Give a formula for the nth square number. Give a formula for the nth pentagonal number. In general, give a formula for the nth r-gonal number.

b. How many numbers can you find that are simultaneously triangular and square? How many numbers can you find that are simultaneously square and pentagonal?

11. Now that you have tried all of the problems, tell us which problem appealed to you the most and why.

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Recommendation Form

This form is to be completed by the applicant's mathematics teacher and mailed directly to: PROMYS, Department of Mathematics and Statistics, Boston University, 111 Cummington Mall, Room 142, Boston, Massachusetts 02215. Please submit this form by May 1, 2013.

For more information, please visit our website at http://www.promys.org, or contact us directly by email, promys@math.bu.edu, or telephone 617-353-2563.

Applicant's Name:____

Teacher's Name and School Address:______________________________

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Telephone:__________________________

Email:_____________________________

How long have you known the applicant?______________________________

In what capacity?____

All activities of PROMYS and the Oxford Masterclasses are held in English. Do you think the applicant's English language skills would enable full participation?________________

Please tell us how you learned of the CMI/PROMYS Alliance.__________________

We would appreciate any comments you can make that might help us determine if the applicant would benefit from the our programs. Because of the highly intense nature of these programs, we are especially interested in the applicant's motivation and potential for sustained hard work on challenging mathematical themes. Since PROMYS and the Oxford Masterclasses are residential programs in which many participants will be living away from home for the first time, we would also like to know your impressions of the applicant's emotional maturity. Please write your comments below (continued on back if necessary), and complete the table on the reverse side by checking the boxes you feel most appropriate. Thank you for your help.

(Next page, please)

Please complete the table below by checking the boxes you feel most appropriately describe the applicant.

	Top 1-2%	Top 5%	Top 10%	Top 25%	Top 50%	Not in top 50%
Interest in mathematics						
Ability to work independently						
Ability to work with others						
Imagination and creativity						
Analytical ability						
Personal initiative						
Perseverance						
Emotional maturity						

Teacher's signature: ______________________________ Date: ________________