

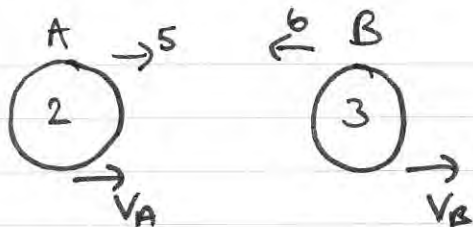
1. Two particles A and B , of mass 2 kg and 3 kg respectively, are moving towards each other in opposite directions along the same straight line on a smooth horizontal surface. The particles collide directly. Immediately before the collision the speed of A is 5 m s^{-1} and the speed of B is 6 m s^{-1} . The magnitude of the impulse exerted on B by A is 14 N s . Find

(a) the speed of A immediately after the collision,

(3)

(b) the speed of B immediately after the collision.

(3)



$$\begin{aligned} \text{Mom A before} &= 10 & \Rightarrow \text{Impulse} &= 14 & \therefore 2v_A &= -4 \\ \text{Mom A after} &= 2v_A & & & & \underline{v_A = -2} \end{aligned}$$

b) CLM $5 \times 2 + 3 \times -6 = 2 \times -2 + 3v_B$

$$\Rightarrow -8 = -4 + 3v_B \Rightarrow 3v_B = -4 \therefore v_B = -\frac{4}{3}$$

2.

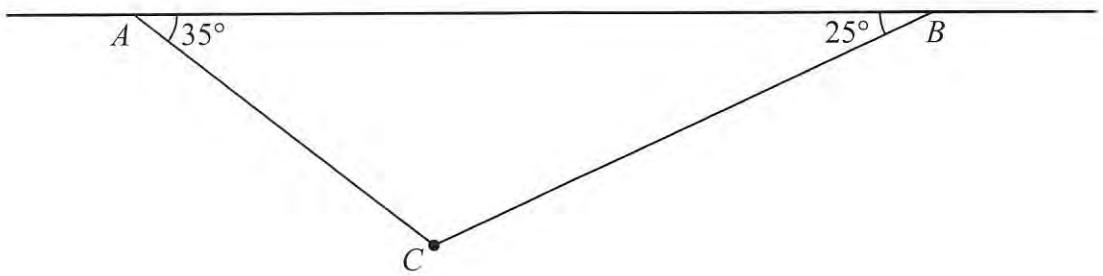
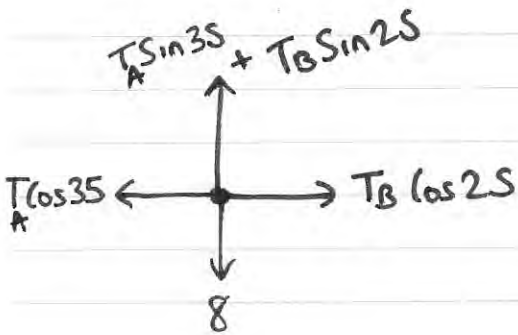


Figure 1

A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC . The other ends, A and B , are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 1. Find

- the tension in the string AC ,
- the tension in the string BC .

(8)



$$\vec{R}_F = 0 \quad \therefore T_A \cos 35^\circ = T_B \cos 25^\circ$$

$$\Rightarrow T_B = \frac{T_A \cos 35^\circ}{\cos 25^\circ}$$

$$R_{\uparrow} = 0 \quad \Rightarrow T_A \sin 35^\circ + T_B \sin 25^\circ = 8$$

$$\Rightarrow T_A \sin 35^\circ + \frac{T_A \cos 35^\circ \sin 25^\circ}{\cos 25^\circ} = 8$$

$$\Rightarrow 0.9555533 T_A = 8 \quad \Rightarrow T_A = \underline{8.37 \text{ N}}$$

$$T_B = \underline{7.57 \text{ N}}$$

3.

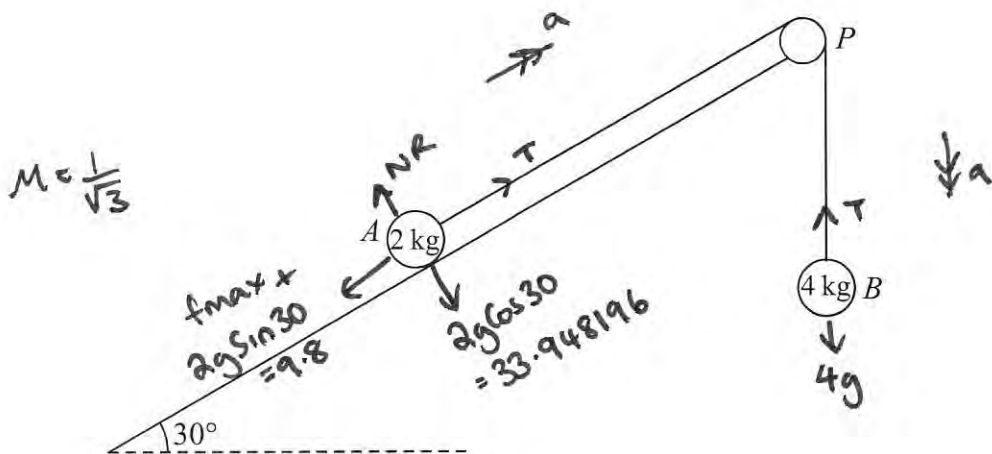


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B , of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P . The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P , as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

$$NR = 33.948196 \Rightarrow f_{\max} = \mu NR = 19.6$$

$$\Rightarrow \text{total force down the plane} = 3g.$$

whole system



$$Rt = ma$$

$$g = 6a$$

$$\therefore a = \frac{1}{6}g$$

$$\textcircled{B} \quad 4g - T = 4a \Rightarrow 4g - T = \frac{4}{6}g$$

$$T = \left(4 - \frac{2}{3}\right)g \quad T = \underline{\underline{\frac{10}{3}g \text{ N}}}$$

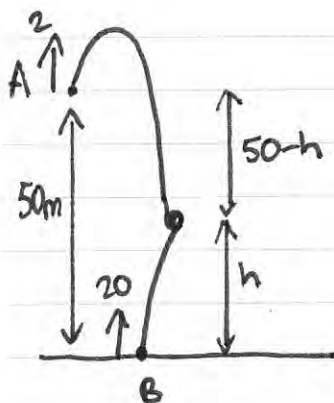
4. At time $t = 0$, two balls A and B are projected vertically upwards. The ball A is projected vertically upwards with speed 2 m s^{-1} from a point 50 m above the horizontal ground. The ball B is projected vertically upwards from the ground with speed 20 m s^{-1} . At time $t = T$ seconds, the two balls are at the same vertical height, h metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of T ,

(5)

(b) the value of h .

(2)



$$\textcircled{A} \quad S = -(50 - h)$$

$$u = 2$$

$$v$$

$$a = -9.8$$

$$t = T$$

$$\textcircled{B} \quad S = h$$

$$u = 20$$

$$v$$

$$a = -9.8$$

$$t = T$$

$$S = ut + \frac{1}{2}at^2 \Rightarrow h - 50 = 2T - 4.9T^2 \quad \textcircled{A}$$

$$h = 20T - 4.9T^2 \quad \textcircled{B} \quad 4.9T^2 = 20T - h$$

$$\Rightarrow h - 50 = 2T + h - 20T \Rightarrow -50 = -18T \therefore T = \frac{25}{9} \text{ sec}$$

$$\text{b) } h = 20\left(\frac{25}{9}\right) - 4.9\left(\frac{25}{9}\right)^2 = \underline{\underline{17.7 \text{ m}}}$$

5.

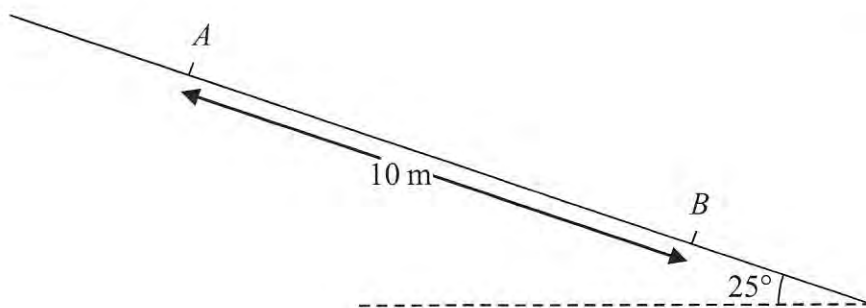


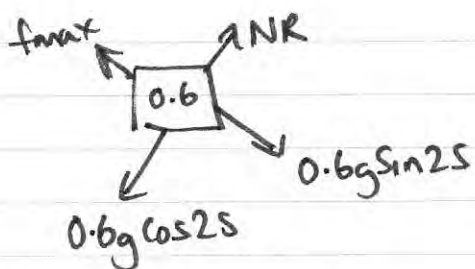
Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B , where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1} . The particle P takes 3.5 s to move from A to B . Find

(a) the speed of P at B , (3)

(b) the acceleration of P , (2)

(c) the coefficient of friction between P and the plane. (5)



$$NR = 5.329089788$$

$$\therefore f_{\max} = 5.3290898\mu$$

$$Rf\downarrow = ma$$

$$\Rightarrow 2.484995 - 5.3290898\mu = 0.6a$$

$$s = 10$$

$$u = 2$$

$$v$$

$$a$$

$$t = 3.5$$

$$s = ut + \frac{1}{2}at^2 \Rightarrow 10 = 7 + \frac{1}{2}a(3.5)^2$$

$$b) \therefore a = \frac{24}{49}$$

$$a) v = u + at \quad v = 2 + \left(\frac{24}{49}\right)\left(\frac{7}{2}\right) = \frac{26}{7}$$

$$c) 2.484995 - 0.6\left(\frac{24}{49}\right) = 5.3290898\mu$$

$$\therefore \mu = 0.41 \text{ (2sf)}$$

6. [In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O .]

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j}) \text{ km}$.

- (a) Find the position vector of S at time t hours.

(2)

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j}) \text{ km h}^{-1}$. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j}) \text{ km}$. The two ships meet at the point P .

- (b) Find the value of n .

(5)

- (c) Find the distance OP .

(4)

$$\text{a) } v = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad s = \begin{pmatrix} -4 \\ 2 \end{pmatrix} + t \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$\text{b) } T = \begin{pmatrix} 6 \\ 1 \end{pmatrix} + t \begin{pmatrix} -2 \\ n \end{pmatrix} \quad \begin{array}{l} 6 - 2t = -4 + 3t \\ 1 = 1 + nt \end{array} \quad \begin{array}{l} 6 - 2t = -4 + 3t \\ 10 = 5t \end{array} \quad \underline{t = 2}$$

$$2 + 3t = 1 + tn \Rightarrow 2 + 6 = 1 + 2n \Rightarrow 2n = 7 \Rightarrow n = 3.5$$

$$\text{c) } p = \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix} \quad \overrightarrow{OP} = \sqrt{2^2 + 8^2} \\ = \underline{\underline{8.25 \text{ km (3sf)}}}$$

7.

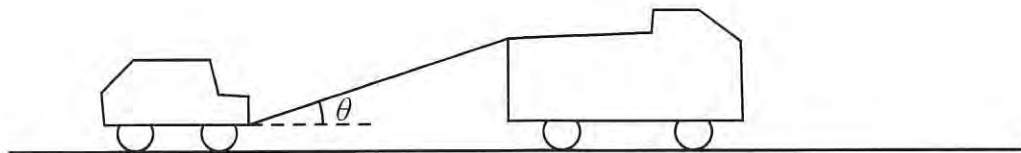


Figure 4

A truck of mass 1750 kg is towing a car of mass 750 kg along a straight horizontal road. The two vehicles are joined by a light towbar which is inclined at an angle θ to the road, as shown in Figure 4. The vehicles are travelling at 20 m s^{-1} as they enter a zone where the speed limit is 14 m s^{-1} . The truck's brakes are applied to give a constant braking force on the truck. The distance travelled between the instant when the brakes are applied and the instant when the speed of each vehicle is 14 m s^{-1} is 100 m.

(a) Find the deceleration of the truck and the car.

(3)

The constant braking force on the truck has magnitude R newtons. The truck and the car also experience constant resistances to motion of 500 N and 300 N respectively. Given that $\cos \theta = 0.9$, find

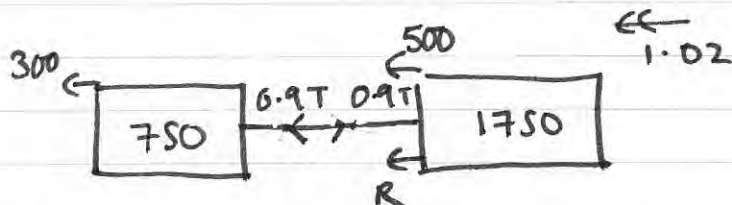
(b) the force in the towbar,

(4)

(c) the value of R .

(4)

$$\begin{aligned}
 s &= 100 & v^2 &= u^2 + 2as & 196 &= 400 + 200a \\
 u &= 20 \\
 v &= 14 & \Rightarrow 200a &= -204 & \Rightarrow a &= \underline{-1.02} \\
 a & & & & & \\
 t & & & & & \therefore \text{deceleration} = \underline{1.02}
 \end{aligned}$$



Note - when breaking tension is THRUST

c) whole system $R + 500 + 300 + 0.9T - 0.9T = 2500 \times 1.02$

$$\Rightarrow R + 800 = 2550 \therefore R = \underline{1750 \text{ N}}$$

b) Car $300 + 0.9T = 750(1.02) \Rightarrow \text{Thrust} = 517 \text{ N}$
~~300 = 750~~ (3sf)

8.

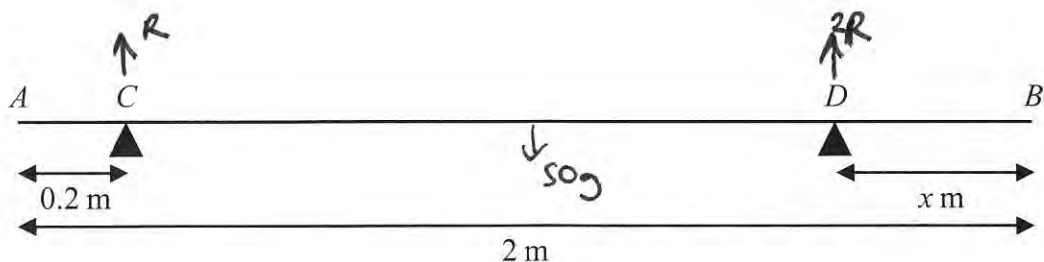


Figure 5

A uniform rod AB has length 2 m and mass 50 kg. The rod is in equilibrium in a horizontal position, resting on two smooth supports at C and D , where $AC = 0.2$ metres and $DB = x$ metres, as shown in Figure 5. Given that the magnitude of the reaction on the rod at D is twice the magnitude of the reaction on the rod at C ,

(a) find the value of x .

(6)

The support at D is now moved to the point E on the rod, where $EB = 0.4$ metres. A particle of mass m kg is placed on the rod at B , and the rod remains in equilibrium in a horizontal position. Given that the magnitude of the reaction on the rod at E is four times the magnitude of the reaction on the rod at C ,

(b) find the value of m .

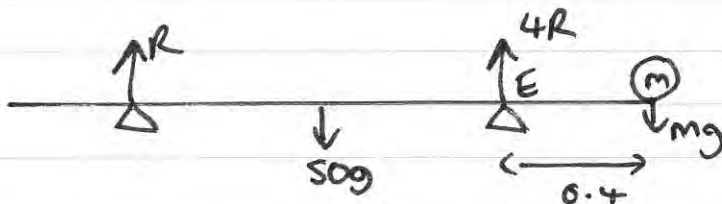
(7)

$$a) \uparrow = \downarrow \Rightarrow 3R = 50g \quad R = \frac{50}{3}g \quad 2R = \frac{100}{3}g.$$

$$B \curvearrowright \quad \frac{100}{3}g \times x + \frac{50}{3}g \times 1.8 = 50g \times 1$$

$$\Rightarrow \frac{100}{3}g \times x = 20g \quad \therefore x = \underline{0.6 \text{ m}}$$

b)



$$B \curvearrowright \quad 4R \times 0.4 + R \times 1.8 = 50g \times 1 \quad \Rightarrow 3.4R = 50g$$

$$\therefore R = \frac{250}{17}g$$

$$\uparrow = \downarrow \Rightarrow 5R = 50g + mg \quad \Rightarrow \frac{1250}{17}g = \frac{850}{17}g + mg$$

$$\therefore m = \frac{400}{17} \quad \underline{23.5 \text{ kg}} \quad (3 \text{ sf})$$