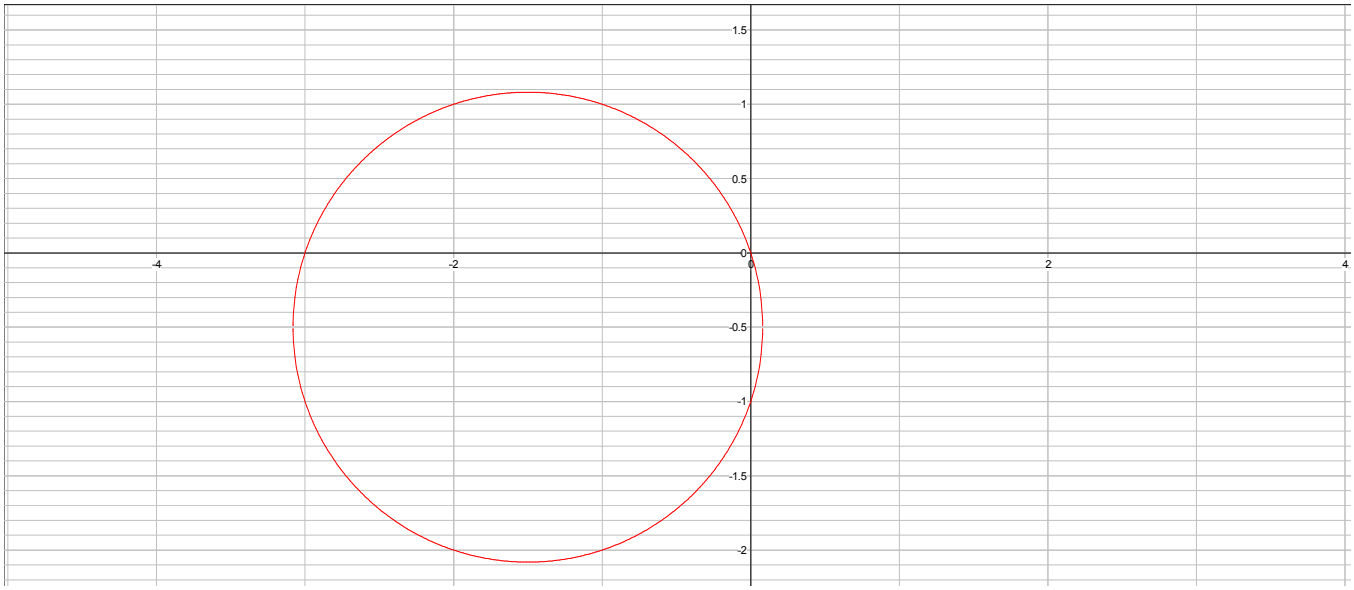
QUESTION 5

Part (i)

$$x^2+y^2+kxy+3x+y=0 \Rightarrow \left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{10}{4} \text{ when } k=0$$

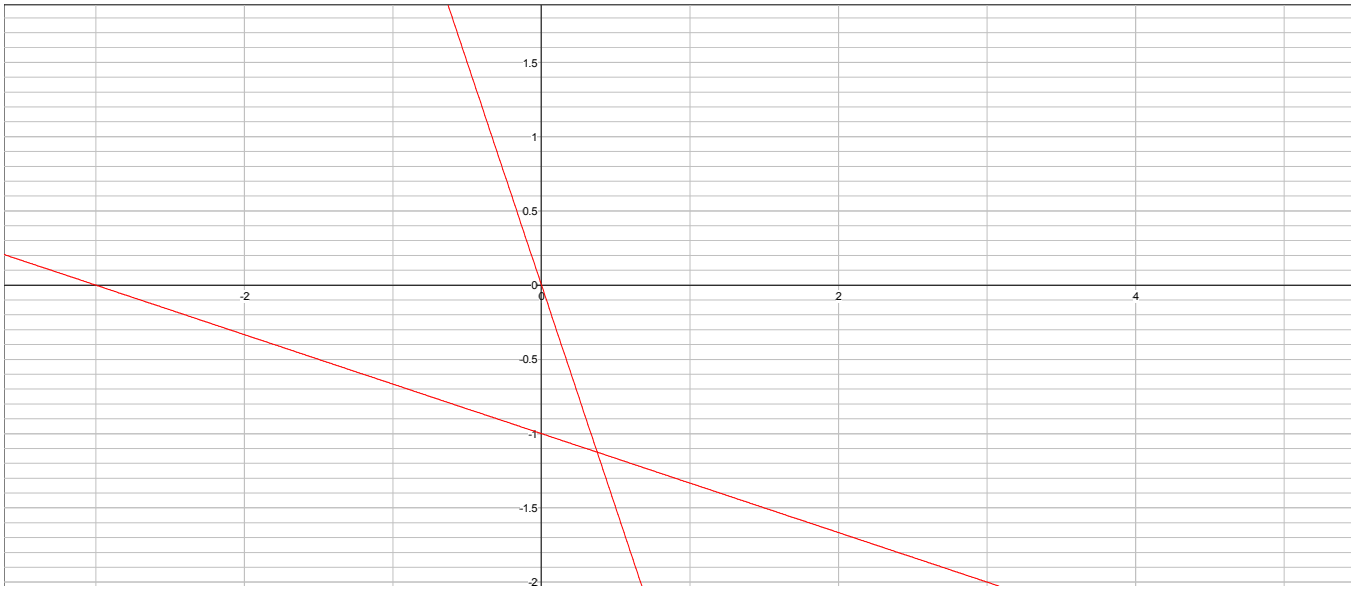


Cuts the coordinate axes at $(0,0),(-3,0),(0,-1)$

Part (ii)

$$x^2+y^2+kxy+3x+y=0 \Rightarrow (3x+y)(x+3y)+3(3x+y)=0 \text{ when } k=\frac{10}{3}$$

$$\Rightarrow (3x+y)(x+3y+3)=0$$



Cuts the coordinate axes $(0,0),(0,-1),(-3,0)$

Part (iii)

With $\theta = 45^0,\ X = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y, Y = -\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y \Rightarrow X+Y = \sqrt{2}y, X-Y = \sqrt{2}x$

Putting $k = 2$ and substituting we have

$$\frac{(X-Y)^2}{2} + \frac{(X+Y)^2}{2} + (X-Y)(X+Y) + \frac{3}{\sqrt{2}}(X-Y) + \frac{1}{\sqrt{2}}(X+Y) = 0$$

$$\Rightarrow 4X^2 + 4\sqrt{2}X - 2\sqrt{2}Y = 0$$

$$\Rightarrow 2X^2 + 2\sqrt{2}X - \sqrt{2}Y = 0$$

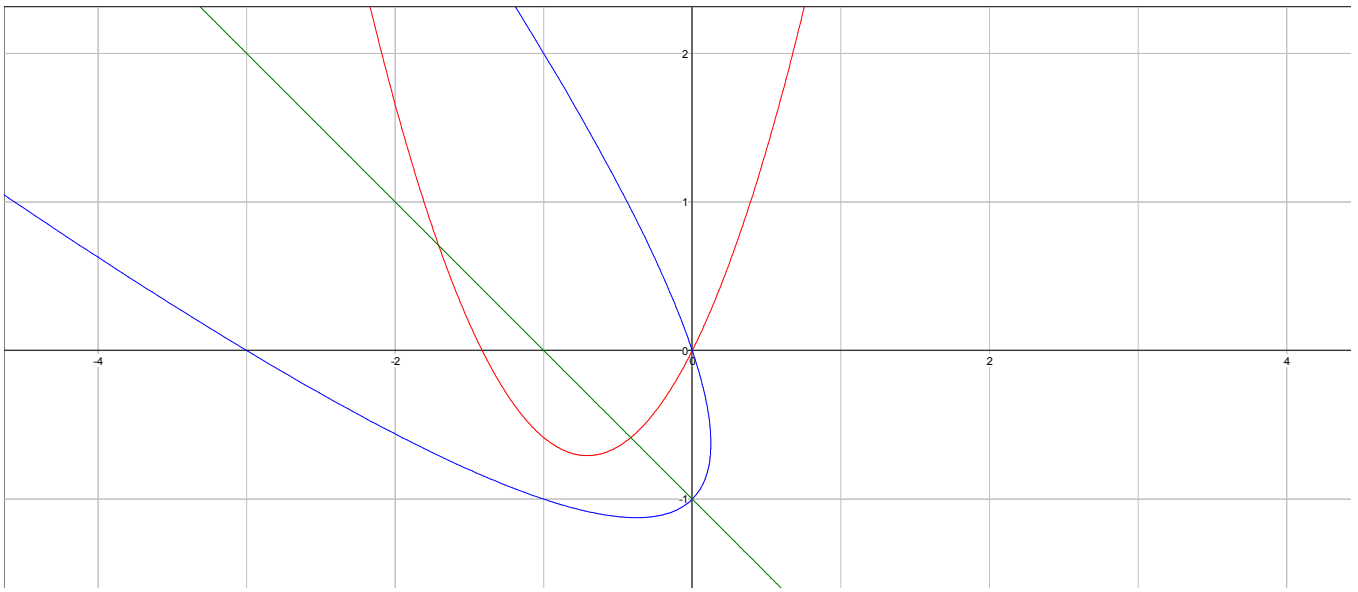
$$\Rightarrow \sqrt{2}Y = \left(\sqrt{2}X + 1\right)^2 - 1$$

Differentiating $\sqrt{2}\frac{dY}{dX} = 2\sqrt{2}\left(\sqrt{2}X+1\right) \Rightarrow \frac{dY}{dX} = 2\left(\sqrt{2}X+1\right) \Rightarrow \frac{dY}{dX} = 0$ when $X = -\frac{1}{\sqrt{2}}$

Clearly the line of symmetry is $X = -\frac{1}{\sqrt{2}}$

When this is rotated through 45^0 anticlockwise about the origin it becomes the line:-

$$y + x + 1 = 0$$



The Red curve is $\sqrt{2}Y = \left(\sqrt{2}X + 1\right)^2 - 1$

The Blue curve is the red curve rotated about the origin through 45^0 anticlockwise so that it is the original curve.

The green curve is the line $y + x + 1 = 0$.
