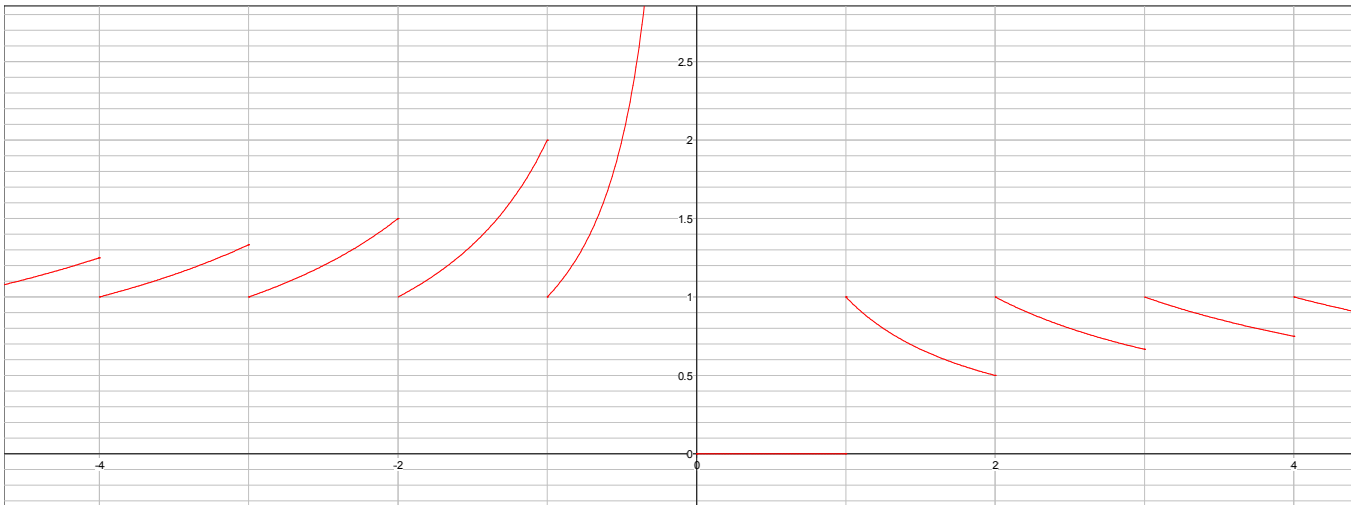
QUESTION 2

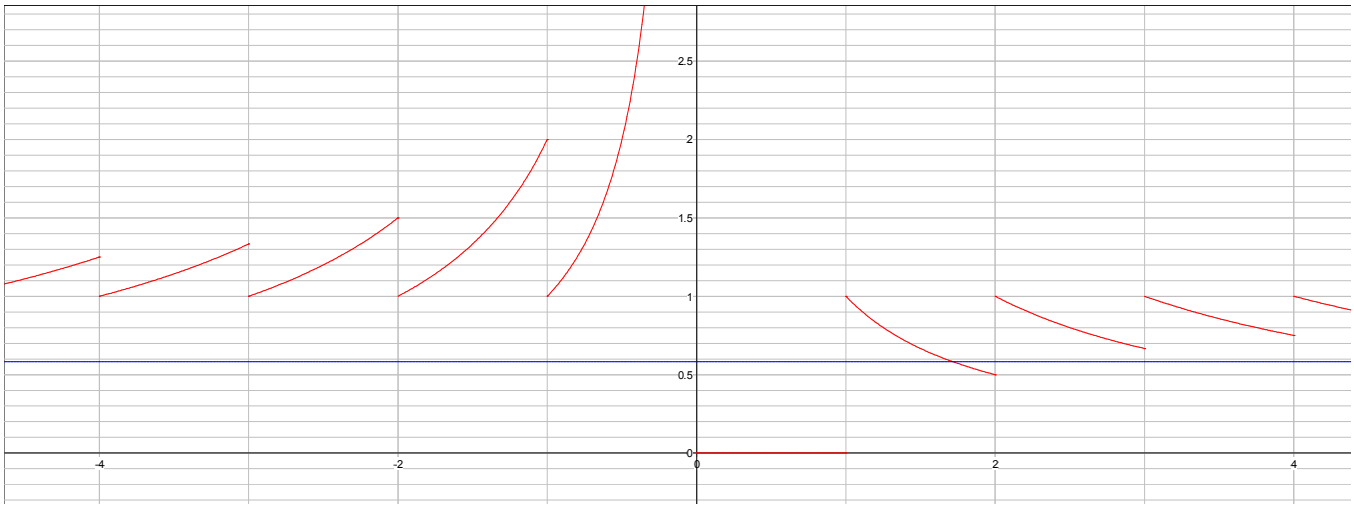
Part (i)



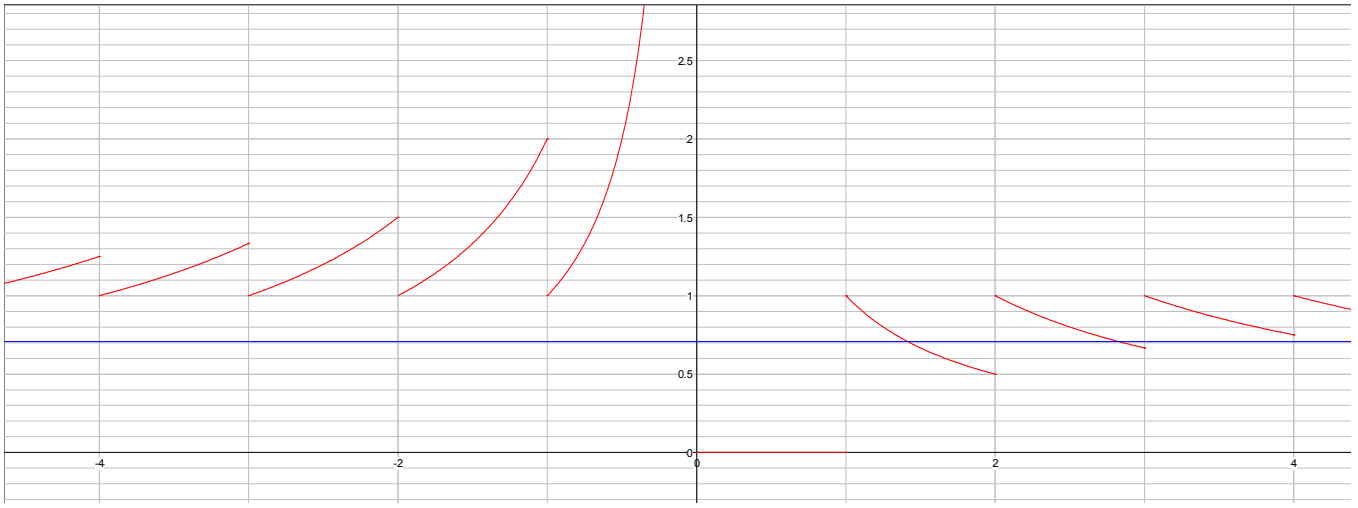
Red curve is $y=\frac{\lfloor x \rfloor}{x}$ Note that $x \neq 0$.

Also the points $(x,1)$ are on the curve for $x \in \mathbb{Z}, x \neq 0$

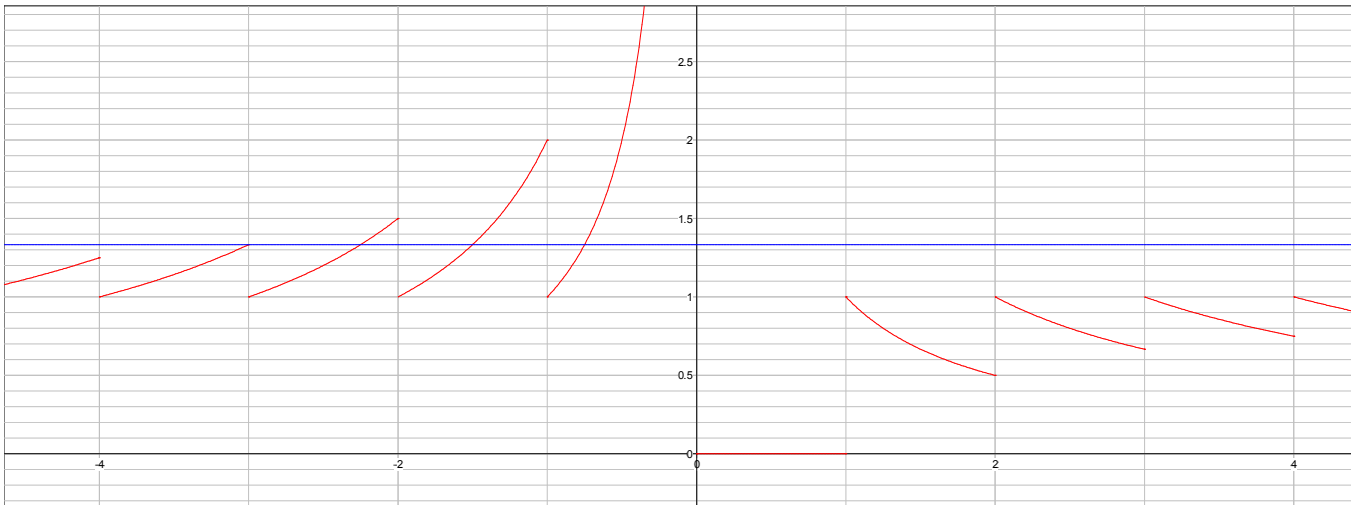
Part (ii)



Red curve is $y=\frac{\lfloor x \rfloor}{x}$, Blue curve is $y=\frac{7}{12}$, Hence $f(x)=\frac{7}{12} \Rightarrow x=\frac{12}{7}$.

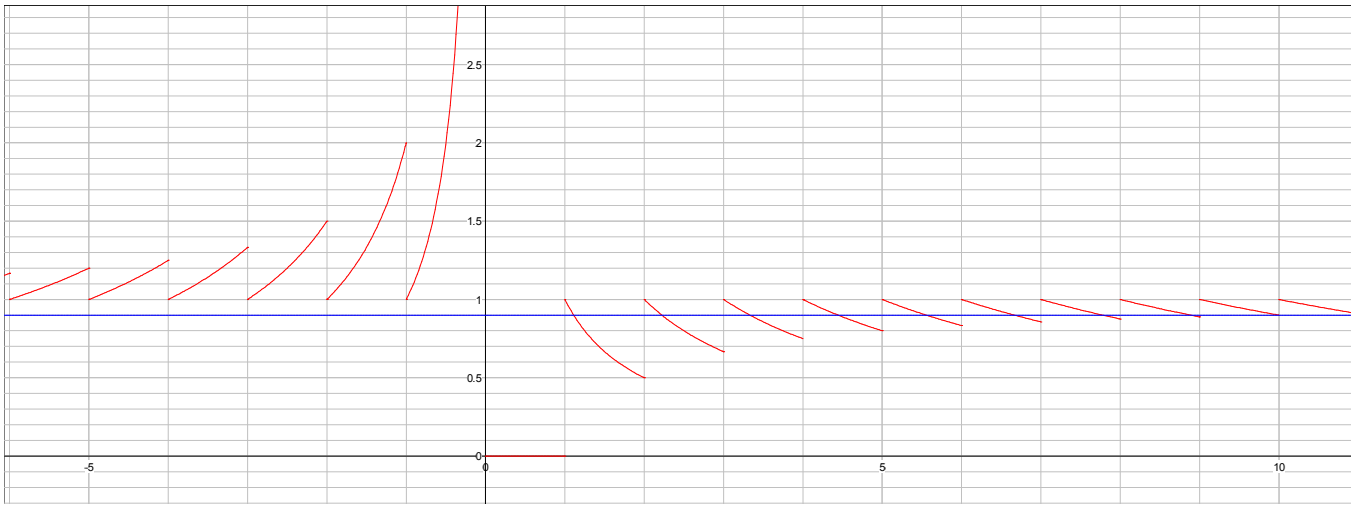


Red curve is $y=\frac{\lfloor x \rfloor}{x}$, Blue curve is $y=\frac{17}{24}$, Hence $f(x)=\frac{17}{24} \Rightarrow x=\frac{24}{17},\frac{48}{17}$.



Red curve is $y=\frac{\lfloor x \rfloor}{x}$, Blue curve is $y=\frac{4}{3}$, Hence $f(x)=\frac{4}{3} \Rightarrow x=-\frac{3}{4},-\frac{3}{2},-\frac{9}{4}$.

Part (iii)



Red curve is $y=\frac{\lfloor x \rfloor}{x}$, Blue curve is $y=\frac{9}{10}$.

Hence the largest root of $f(x)=\frac{9}{10}$ is $x=\frac{80}{9}$.

If $c<1$ for the equation $f(x)=c$ to have exactly n roots then:-

$$\frac{n}{n+1}<c\leq\frac{n+1}{n+2}$$

If $c>1$ for the equation $f(x)=c$ to have exactly n roots then:-

$$\frac{n+1}{n}\leq c<\frac{n}{n-1}$$

For $c=0$ or 1 there are always an infinite number of solutions so that it is impossible to specify n .