

DERIVATIVE TEST FOR A POINT OF INFLECTION.

THEOREM:

**If the graph of $f(x)$ has an inflection point at x_0 , and $f''(x_0)$ exists in an open interval containing x_0 , and f'' is continuous at x_0 ,
then $f''(x_0) = 0$**

However, the converse of this is not true - i.e. it's true that if x_0 is an inflection point, then $f''(x_0) = 0$

it's not true (generally) that if $f''(x_0) = 0$ then $f(x_0)$ is an inflection point.

Here is how you can tell a point is DEFINITELY an inflection point:

**If $f''(x_0) = 0$ and $f^{(n+1)}(x_0) \neq 0$ for some $n=2k$, $k \in \mathbb{Z}$
then, there is an inflection point at $(x_0, f(x_0))$**

OR:

Let $f(x)$ be a real valued differentiable function, on an interval within $\mathbb{R}$, an $n \geq 1$ an integer.

$$\text{If } f'(c) = f''(c) = f'''(c) = \dots = f^{(n)}(c) = 0$$

and $f^{(n+1)}(c) \neq 0$, then either:

1) n is odd and:

$$f^{(n+1)}(c) < 0 \Rightarrow x = c \text{ is a relative maximum.}$$

or

$$f^{(n+1)}(c) > 0 \Rightarrow x = c \text{ is a relative minimum.}$$

OR:

2) n is even, and:

$$f^{(n+1)}(c) < 0 \Rightarrow x = c \text{ is a strictly decreasing point of inflection.}$$

or

$f^{(n+1)}(c) > 0 \Rightarrow x = c$ is a strictly increasing point of inflection.