OXFORD ENTRANCE PRACTICE

(1) Prove that the sum of the first n terms of the artithmetical progression

$$a + (a + d) + (a + 2d) + ...,$$

 $\frac{1}{2}n\{2a + (n - 1)d\}.$

is

$$\frac{2^{n}(2u + (n - 1))}{2^{n}(2u + (n - 1))}$$

Hence, or otherwise, show that

$$1^{2} - 2^{2} + 3^{2} - 4^{2} + \dots + (2n - 1)^{2} - (2n)^{2} = -n(2n + 1).$$

Deduce the sums of the series

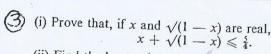
(a)
$$1^2 - 2^2 + 3^2 - 4^2 + ... + (2n-1)^2 - (2n)^2 + (2n+1)^2$$
,

and /

(b)
$$21^2 - 22^2 + 23^2 - 24^2 + ... + 39^2 - 40^2$$
.

The point O is the origin of coordinates, A is the point (— 5, 5) and C is the point (7, 1). Find, by calculation, the coordinates of the point B such that both AB = BC and also angle OAB is a right

Prove that 0,4,B,C lie on a circle and find



(ii) Find the least value of

 $f(x) \equiv \sqrt{(1-x)} + \sqrt{(4-x)},$ given that x and f(x) are real.

(iii) If x and y are real, sketch the whole of the curve $y = \sqrt{1-x^2} + \sqrt{4-x^2}$.

The symbol $\sqrt{(u)}$ denotes the non-negative value of the square root of u.]

(4) Sketch the curve

$$y = \frac{(x-1)(x-4)}{(x-2)(x-3)}$$

and also, using this cuve if you wish and explaining your argument carefully, the curve

$$y^2 = \frac{(x-1)(x-4)}{(x-2)(x-3)}.$$

Indicate in a diagram the positions of the points A_0 , A_1 , A_2 , A_3 whose coordinates referred to rectangular Cartesian axes Ox, Oy are

$$A_0(1, 1), A_1(\frac{1}{2}, \frac{1}{3}), A_2(\frac{1}{4}, \frac{1}{1}), A_3(\frac{1}{8}, \frac{1}{27}).$$
Consider the $(n + 1)$ points $A_1(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ where

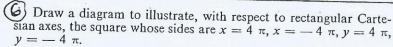
Consider the (n + 1) points $A_p(2^{-p}, 3^{-p})$, where

$$p = 0, 1, 2, ..., n.$$

(i) Find the limit as $n \rightarrow \infty$ of the sum of the areas of the n rectangles having sides parallel to Ox, Oy and opposite vertices at adjacent points A_p , A_{p+1} [The area of the first such rectangle, with p = 0, is $\frac{1}{3}$.

(ii) Find the limit as $n \to \infty$ of the sum of the areas of the circles circumscribing these rectangles. [The area of the first such circle is

 $\frac{25}{144}\pi$.]



Indicate all the points inside the square for which

$$\sin y = \sin x$$

showing that they consist of two isolated points (at two of the vertices) and a number of straight lines of total length 64 $\pi\sqrt{2}$. [The argument giving this number must be clearly stated.]

