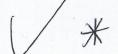
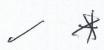
Given that  $2y = a^x + a^{-x}$ , where a > 1, x > 0, prove that  $a^x = y + \sqrt{(y^2 - 1)}$ . If, further,  $2z = a^{3x} + a^{-3x}$ , prove that  $z = 4y^3 - 3y$ .



Draw a sketch-graph of the curve whose equation is

 $y = x^2(2-x)$ . Hence, or otherwise, draw a sketch-graph of the curve whose equation is

 $y^2 = x^2(2-x),$ indicating briefly how the form of the curve has been derived.



(3) Find values of x and y which are positive integers and which satisfy simultaneously the inequalities

$$2x + 3y > 12$$
,  $x^2 + y^2 < 6x + 4y$ .



(A) Prove that the equation

 $x^2+y^2-7x-qy+6=0$  represents the circle through the points (1, 0), (6, 0) having its centre at the point  $A(\frac{7}{2}, \frac{1}{2}q)$ ; and find the equation of the circle through the points (0, 2), (0, 3) having its centre at the point  $B(\frac{1}{2}p, \frac{5}{2})$ . Assuming that the circles intersect, prove that

(p-7)x = (q-5)yis the equation of the chord common to the two circles. Discuss briefly the case p = 7, q = 5.

Prove also that, if the circles are so related that the sum of the squares of their radii is equal to  $AB^2$ , then 7p+5q=24.



(S) Sketch the curve

$$y = x(x-1)(x-\lambda)$$

for each of the cases

$$\lambda < 0, \quad \lambda = 0, \quad 0 < \lambda < 1, \quad \lambda = 1, \quad \lambda > 1.$$

[Only the general shape is required. In particular, you need not work out exact positions or values for the maxima and minima of y.]

The turning points of the curve are at  $A(x_1, y_1)$  and  $B(x_2, y_2)$ . Prove that

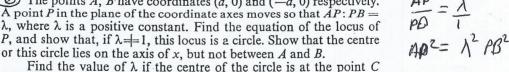
$$x_1 + x_2 = \frac{2}{3}(\lambda + 1), x_1 x_2 = \frac{1}{3}\lambda,$$

and deduce that the gradient of the line AB is negative for all values of  $\lambda$ .



(a, 0) The points A, B have coordinates (a, 0) and (-a, 0) respectively. A point  $\hat{P}$  in the plane of the coordinate axes moves so that  $\hat{AP}$ : PB = $\lambda$ , where  $\lambda$  is a positive constant. Find the equation of the locus of P, and show that, if  $\lambda = 1$ , this locus is a circle. Show that the centre

Find the value of  $\lambda$  if the centre of the circle is at the point C on the axis of x, where CB:CA = 3:5.





 $\mathcal{T}$  Find all integer values of x and y that satisfy simultaneously the inequalities

$$x+2y \geqslant 2, 
-x+3y \leqslant 3, 
3x-4y \leqslant 6.$$



(8) Solve completely the simultaneous equations

$$x+y+z=3,$$
  
 $x+2y+4z=7,$   
 $x+ky+k2z=1+k+k3$ 

 $x+ky+k^2z = 1+k+k^3$ . Your solution should give (i) expressions for x,y,z for a general value of k; (ii) formulae giving all values of x,y,z for those values of k for which the solution is not unique; (iii) any value of k for which the equations have no solution.

(Be careful to copy the third equation correctly.)

