1. Using $\ln f(x) \to f^{'}(x)/f(x)$:

d/dx (ln(cosec x + cot x)) = (-cosec x cot x - cosec^2 x)/(cosec x + cot x)

-cosec x ((cosec x + cot x))/((cosec x + cot x)) = - cosec x.

2. a) f(-3/2) = (9/4+p)(0) + 3 = 3

b) f(2) = (4+p)(7) + 3 = 24 $\Rightarrow$ 28 + 7p = 21 $\Rightarrow$ p = -1.

c) f(x) = $2x^3 + 3x^2 - 2x = x(2x^2 + 3x - 2) = x(2x - 1)(x + 2)$

3. a) $r = 13 \Rightarrow (x - 5)^2 + (y - 13)^2 = 169$

b) $x^2 - 10x + y^2 - 26y + 25 = 0 \Rightarrow 2x - 10 + dy/dx(2y - 26) = 0.$

$\Rightarrow dy/dx = 10 - 2x/2y - 26 = (5 - x)/(y - 13)$

$\Rightarrow$ equation of the tangent is given by $y - 1 = -5/-12(x - 10)$

$\Rightarrow 12y - 12 = 5x - 50 \Rightarrow 12y - 5x + 38 = 0$

4. $u = 1 + sinx \Rightarrow du/dx = cosx \Rightarrow dx = du/cosx$

$\Rightarrow \int sinxcosx(1 + sinx)^5 dx = \int (u - 1)u^5 = u^7/7 - u^6/6$

$= (1 + sinx)^7/7 - (1 + sinx)^6/6 = 1/42(1 + sinx)^6(6(1 + sinx) - 7) = 1/42(1 + sinx)^6(6sinx - 1) + C$

5. a) $A = 1, B = 2$

b) $3 + 5x/(1 + 3x)(1 - x) = 1/(1 + 3x) + 2/(1 - x) = (1 + 3x)^{-1} + 2(1 - x)^{-1} = 1 - 3x + 9x^2 + 2(1 + x + x^2) = 3 - x + 11x^2$

c) No, the binomial expansion assumes that $|3x| \leq 1$, i.e $|x| \leq 1/3$.

6. a) 4 = 2 sect $\Rightarrow t = \pi/3 \Rightarrow x = 3tsint = \pi\sqrt{3}/2$

b) $\int$ ydx = $\int$ 2 sect dx/dt dt.

$dx/dt = 3(sint + tcost)$

$\Rightarrow \int$ 2 sect dx/dt dt = $\int$ $6tant + 6t = 6 \int tant + t$. Limits are values of t, i.e 0 and $\pi/3$.

c) $6\int tant + t = 6(ln(sec(t)) + t^2/2)$. Putting in limits gives $6(ln(2) + \pi^2/18) = 6ln(2) + \pi^2/3$

7) a) A = πr^2 = 16π(1-2$e^{-\lambda t} + e^{-2\lambda t}$)

$\Rightarrow dA/dt = 32\pi\lambda e^{-\lambda t} - 32\pi\lambda e^{-2\lambda t} = 32\pi\lambda(e^{-\lambda t} - e^{-2\lambda t})$

b) $A^{-3/2}dA = t^{-2}dt.$

$\Rightarrow -2A^{-1/2} = -1/t + C \Rightarrow 2/A^{1/2} = 1/t + C$

$A = 1$, $t = 1 \Rightarrow C = 1.$

$\Rightarrow 2/A^{1/2}$ = (t+1)/t $\Rightarrow (2t/(t + 1))^2 = A$

c) $A = 4(t/(t + 1))^2 < 4$

8. a) r_1 = (9-8t)i + (2-3t)j + (4+5t)k, r_2 = (s-16)i + (α-4s)j +(9s+10)k.

Set the i and k components equal: 9-8t = s-16, so s = 25-8t.

9s+10 = 4+5t $\Rightarrow$ 225-72t+10 = 4+5t. $\Rightarrow$ 231 = 77t $\Rightarrow$ t = 3, s = 1.

So α - 4 = 2-3t = -7, hence α = -3.

b) t = 3, so A = -15i-7j+19k.

c) Using scalar product: (-8i-3j+5k).(i-4j+9k) = 98 cos α

$\Rightarrow$ -8+12+45 = 49 = 98 cos α. Hence cos $\alpha = 1/2 \Rightarrow \alpha = 60^o$