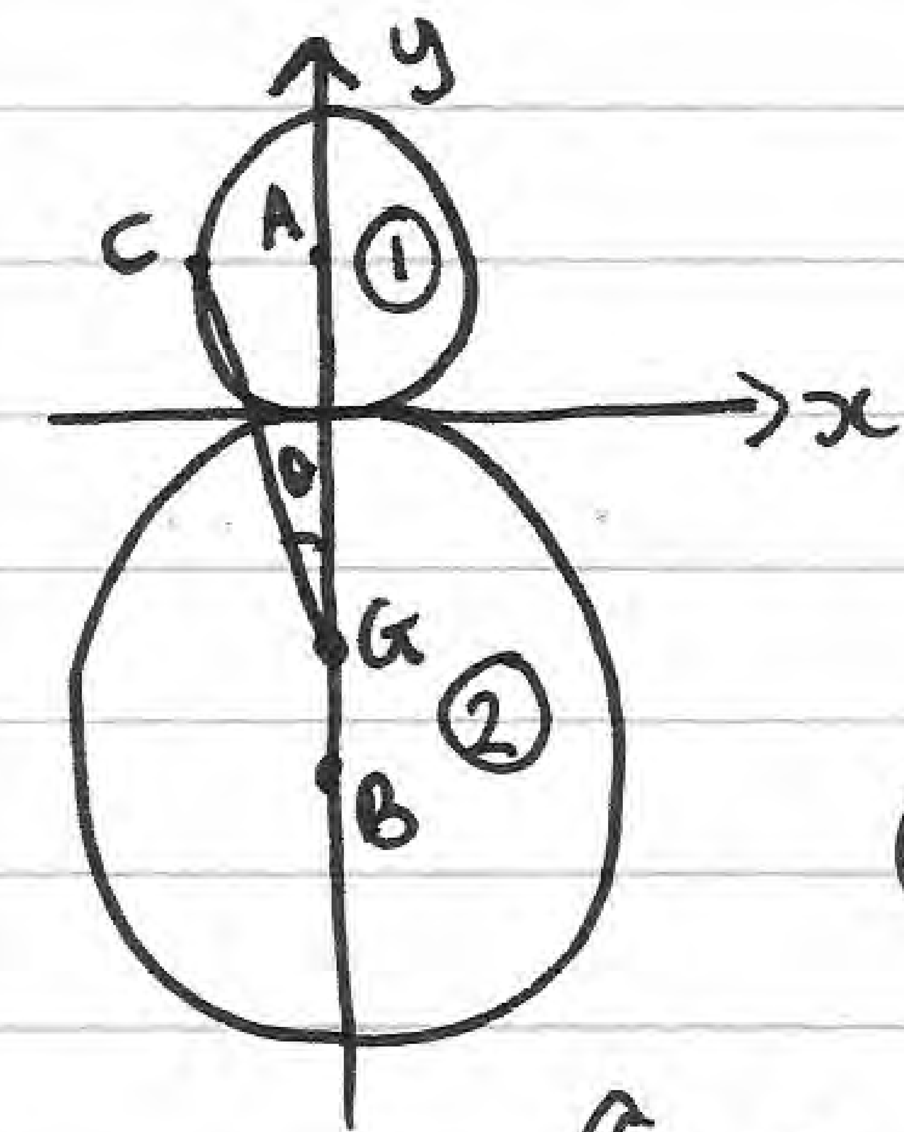
M2 JUNE 01

1) $r = (t^2+2t)i + (t-2t^2)j$

$v = \frac{dr}{dt} = (2t+2)i + (1-4t)j$

$a = \frac{dv}{dt} = 2i - 4j \Rightarrow |a| = \sqrt{2^2+4^2} = \underline{2\sqrt{5}\ ms^{-2}}$

2)



μ = mass per unit cm^2

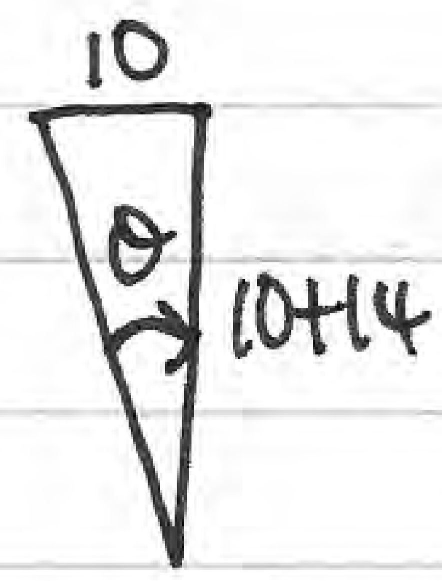
① $M = 100\pi\mu$ $g_1(0,10)$
② $M = 400\pi\mu$ $g_2(0,-20)$
①+② $M = 500\pi\mu$ $G(0,\bar{y})$

$\cancel{100\pi\mu g} \times 10 + \cancel{400\pi\mu g} \times -20 = \cancel{500\pi\mu g}\,\bar{y}$

$\Rightarrow 1000 - 8000 = 500\bar{y} \Rightarrow \bar{y} = -14$

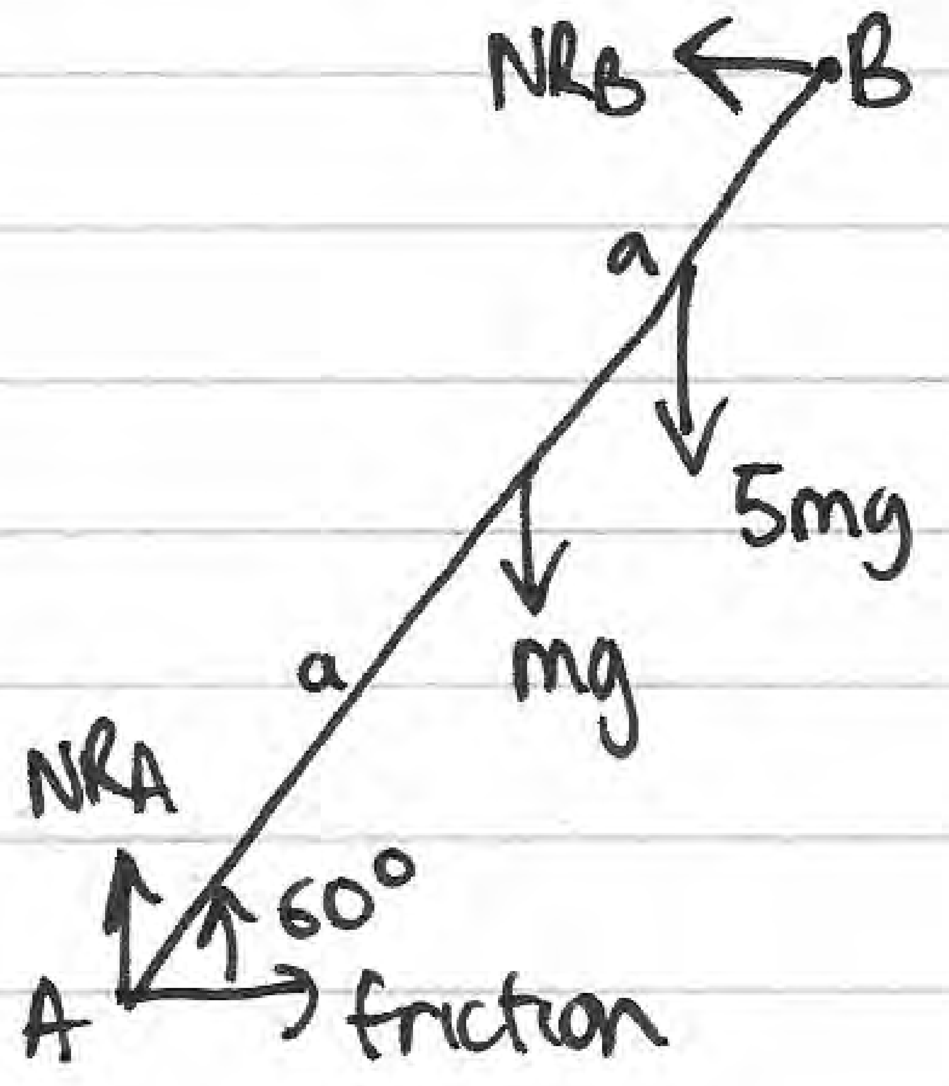
$\Rightarrow$ 6cm above B

b)



$\theta = \tan^{-1}\left(\frac{10}{24}\right) = \underline{22.6°}$

3)



$M = \frac{1}{2}$

at greatest value of μ, ladder will be in limiting equilibrium

$\Rightarrow$ friction $= f_{max} = \mu NRA = \frac{1}{2} NRA$

$R_f \uparrow = \downarrow \Rightarrow NRA = 6mg \Rightarrow f_{max} = 3mg.$

$R_f \overrightarrow{} = 0 \Rightarrow NRB = f_{max} = 3mg.$

A↺ $\cancel{mg} \times a\cos 60 + 5\cancel{mg} \times \mu a\cos 60 = 3\cancel{mg} \times 2a\sin 60$

$\frac{1}{2}\cancel{\alpha} + \frac{5}{2}\mu\cancel{\alpha} = 3\sqrt{3}\cancel{\alpha}$ $\quad \mu = \frac{6\sqrt{3}-1}{5} = \underline{1.88\ (3sf)}$

4) Mom before + Impulse = Mom after

$\Rightarrow 0.1u + 3.5i + 3j = 0.1(10i + 25j)$

$\Rightarrow 0.1u = -2.5i - 2.5j$ $\qquad \Rightarrow u = -25i + 25j \ ms^{-1}$

b) $u\uparrow = 25$ $\qquad v^2 = u^2 + 2as$

$v\uparrow = 0$ $\qquad 0 = 25^2 - 19.6s \qquad s \Rightarrow 31.9m$

$a\uparrow = -9.8$ $\qquad \Rightarrow$ 32.9m above ground.

c) $s\uparrow = 0$ $\qquad s = ut + \frac{1}{2}at^2 \Rightarrow 0 = 25t - 4.9t^2$

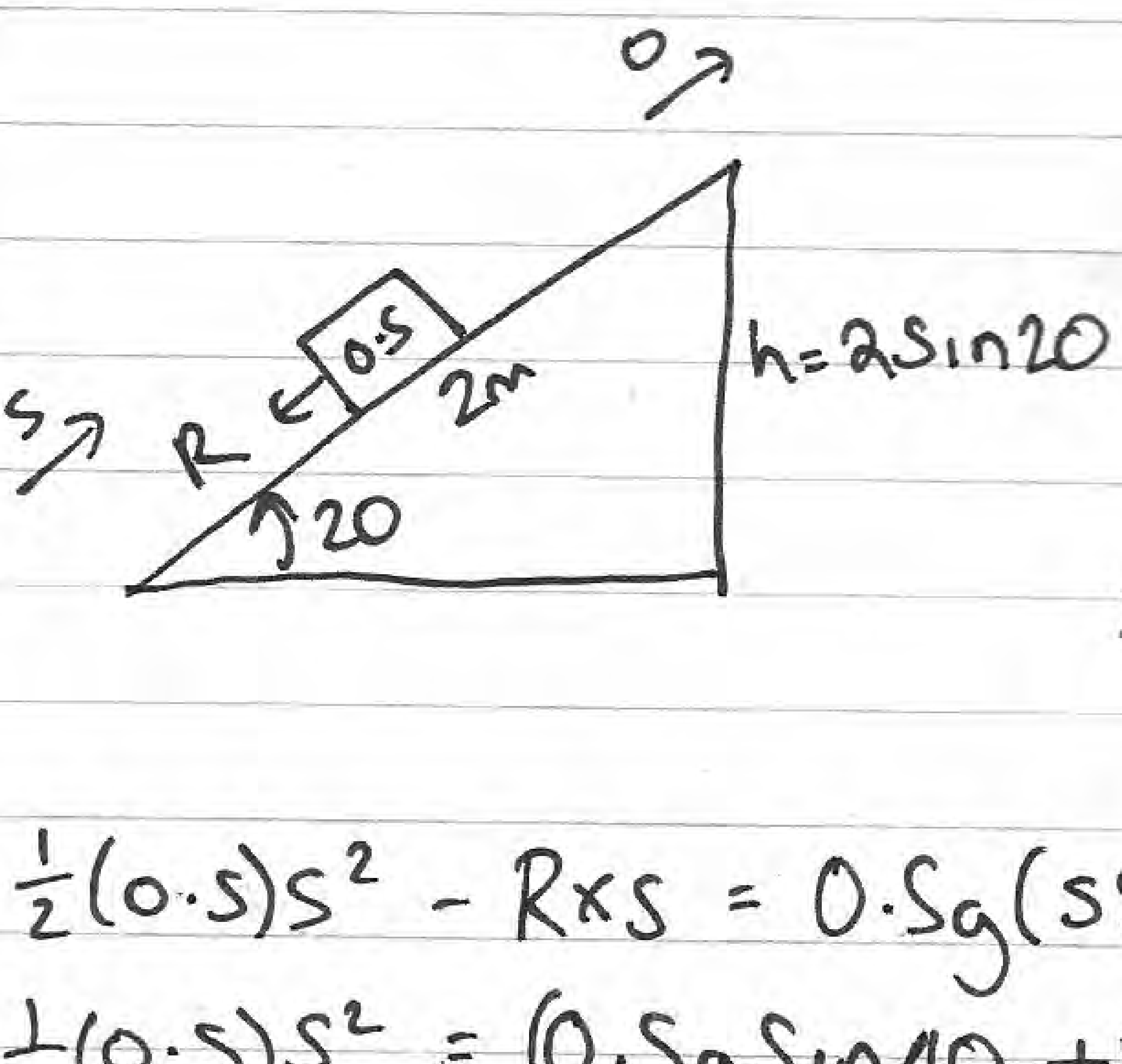
$0 = t(25 - 4.9t)$

$\Rightarrow t = 5.10...$

$\vec{H}$ Vel = 10 $\quad t = 5.10..$

$x = 10 \times 5.10.. \qquad x = 51m$ (2sf)

5)



KE_A - Wd against R = PE_B

$\frac{1}{2}(0.5)5^2 - R\times 2 = \cancel{0.5}g(\cancel{2}\sin 20)$

$2R = \frac{25}{4} - g\sin 20 \Rightarrow R = 1.45N$ (3sf)

b) $\frac{1}{2}(0.5)5^2 - R\times s = 0.5g(s\sin 40)$

$\frac{1}{2}(0.5)5^2 = (0.5g\sin 40 + R)s \qquad \Rightarrow s = 1.36m$ (3sf)

6) A (2m) $\rightarrow 2u$ $\qquad$ B (4m) $\rightarrow u$ $\qquad e = \frac{1}{2} = \frac{V_B - V_A}{u} \Rightarrow u = 2V_B - 2V_A$

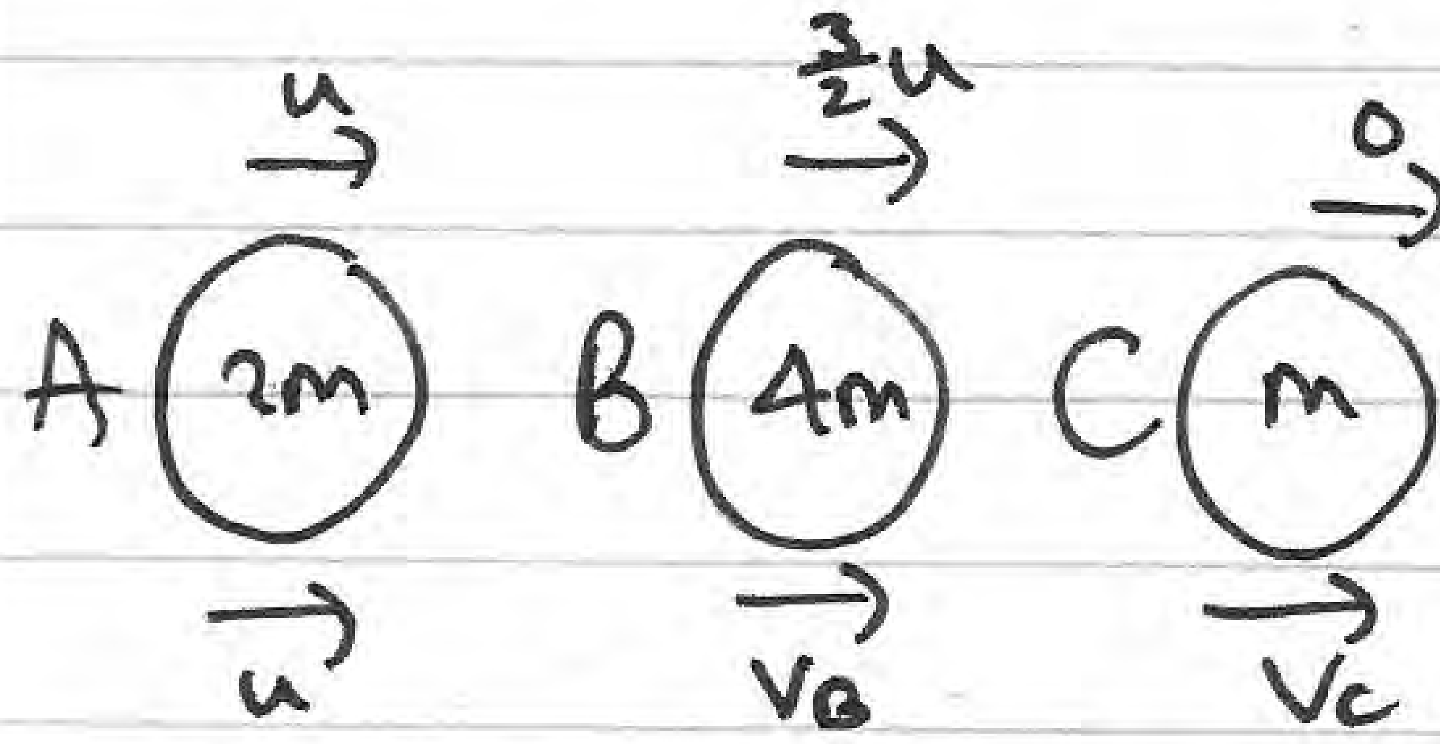
$\rightarrow V_A \qquad \rightarrow V_B \qquad 2V_B = u + 2V_A$

CLM $\Rightarrow 4mu + 4mu = 2mV_A + 4mV_B$

$\Rightarrow 8mu = 2mV_A + 2mu + 4mV_A \Rightarrow 8\cancel{m}u = 6\cancel{m}V_A + 2\cancel{m}u$

$\Rightarrow 6u = 6V_A \quad \Rightarrow \quad V_A = u$

$\Rightarrow \quad 2V_B = 3u \Rightarrow V_B = \frac{3}{2}u$ #



$e = \dfrac{V_C - V_B}{\frac{3}{2}u} \Rightarrow 3eu = 2V_C - 2V_B$

CLM $\Rightarrow 6\cancel{m}u = 4\cancel{m}V_B + \cancel{m}V_C \quad \Rightarrow V_C = 6u - 4V_B$

$\Rightarrow 3eu = 12u - 8V_B - 2V_B \quad \Rightarrow \quad 10V_B = 12u - 3eu$

$\Rightarrow V_B = \frac{1}{10}u(12 - 3e)$

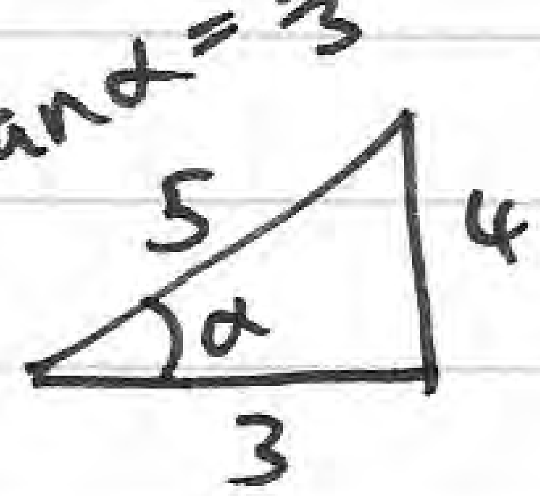
If there are no further collision $V_B \geqslant u$

$\frac{1}{10}u(12-3e) \geqslant u \quad \Rightarrow 12 - 3e \geqslant 10 \quad \Rightarrow \quad 2 \geqslant 3e$

$\Rightarrow e \leqslant \frac{2}{3}$

$0 < e \leq \frac{2}{3}$

7) $\tan\alpha = \frac{4}{3}$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

$V\uparrow$

$u\uparrow = 23.75 \times \frac{4}{5}$

$s\uparrow = -2.4$

$a\uparrow = -9.8$

$s = ut + \frac{1}{2}at^2 \Rightarrow -2.4 = 19t - 4.9t^2 \quad \Rightarrow 4.9t^2 - 19t - 2.4 = 0$

$t = \dfrac{19 + \sqrt{19^2 - 4(4.9)(-2.4)}}{9.8} \quad \Rightarrow t = 4\text{ sec}$

b) $V = \int a\,dt = -\frac{1}{12}t^3 + C \qquad V = 18, t = 0 \Rightarrow C = 18$

$V = -\frac{1}{12}t^3 + 18$

when $t = T$ $V = 0$ $0 = -\frac{1}{12}T^3 + 18$

$\Rightarrow T^3 = 216 \Rightarrow \underline{T = 6\text{ sec}}$

d) $\vec{H}$ $\vec{u} = 23.75 \times \frac{3}{5} = 14.25$

$t = 4$

$x = 14.25 \times 4 = 57\text{m}$ $AC = 57\text{m}.$

$V = -\frac{1}{12}t^3 + 18$ $S = \int v\,dt = -\frac{1}{48}t^4 + 18t + C$

$S = 0, t = 0 \Rightarrow C = 0$ $\Rightarrow S = -\frac{1}{48}t^4 + 18t$

$t = 4$ $S = 66\frac{2}{3}\text{m}.$

Lorry is $66\frac{2}{3}$m from A when C hits the ground

$\therefore$ Lorry is $66\frac{2}{3} - 57 = \underline{9\frac{2}{3}\text{m}}$ ahead of C