

M2 JUNE 02

1) $v = (3t-2)i - 5tj$

a) $a = \frac{dv}{dt} = 3i - 5j$

b) $s = \int v dt = (\frac{3}{2}t^2 - 2t + C_1)i - (\frac{5}{2}t^2 + C_2)j$

$t=0, s=3i \Rightarrow C_1=3, C_2=0$

$t=2 \quad s = 5i + 10j \Rightarrow \text{dist} = \sqrt{5^2 + 10^2} = \underline{5\sqrt{5}m}$

2) $t=3 \quad a = 4t - t^2$

$v = \int a dt = 2t^2 - \frac{1}{3}t^3 + C$

$t=0, v=0 \Rightarrow C=0 \quad v = 2t^2 - \frac{1}{3}t^3$

$t=3, v = \underline{9 \text{ ms}^{-1}}$

b) $t=6 \quad a = \frac{27}{t^2} \quad v = \int a dt = \int 27t^{-2} dt$

$v = -27t^{-1} + C \Rightarrow v = -\frac{27}{t} + C$

$t=3, v=9 \Rightarrow 9 = -\frac{27}{3} + C \Rightarrow C=18$

$\Rightarrow v = 18 - \frac{27}{t} \quad t=6 \Rightarrow v = \underline{13.5 \text{ ms}^{-1}}$

3) $KE_A + PE_A + W_{\text{d by Cyclist}} - W_{\text{d against Res}} = KE_B + PE_B$

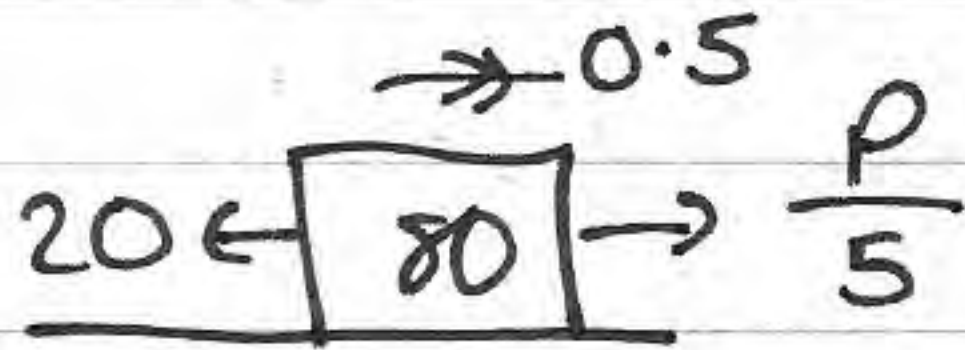
$\therefore \text{loss in KE} + \text{loss in PE} + W_{\text{d by Cyclist}} = W_{\text{d against Res}}$

$\frac{1}{2}(80)(8^2 - 5^2) + 80g(20-12) + W_{\text{d by C}} = 20 \times 500$

$$\Rightarrow 1560 + 6272 + \text{wd by C} = 10000$$

$$\therefore \text{wd by C} = 2168 \text{ J}$$

b)



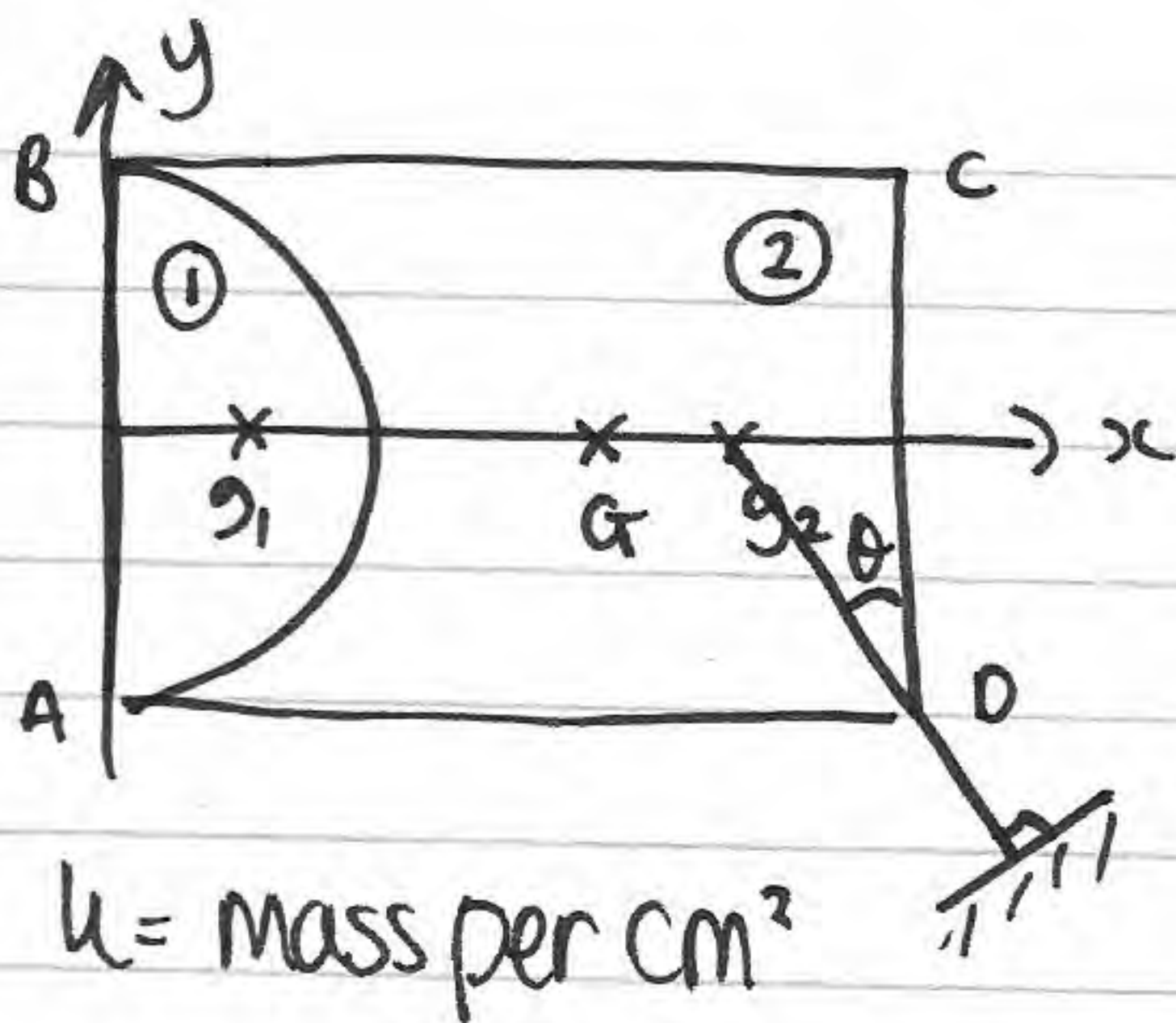
$$\vec{Rf} = ma$$

$$\frac{P}{5} - 20 = 80 \times 0.5$$

$$\Rightarrow \frac{P}{5} = 60$$

$$\Rightarrow \underline{P = 300 \text{ W}}$$

4)



$$M_1 = \frac{25}{2}\pi u \quad g_1 = \left(\frac{20}{3\pi}, 0\right)$$

$$M_2 = \left(100 - \frac{25}{2}\pi\right)u \quad g_2 = (\bar{x}, 0)$$

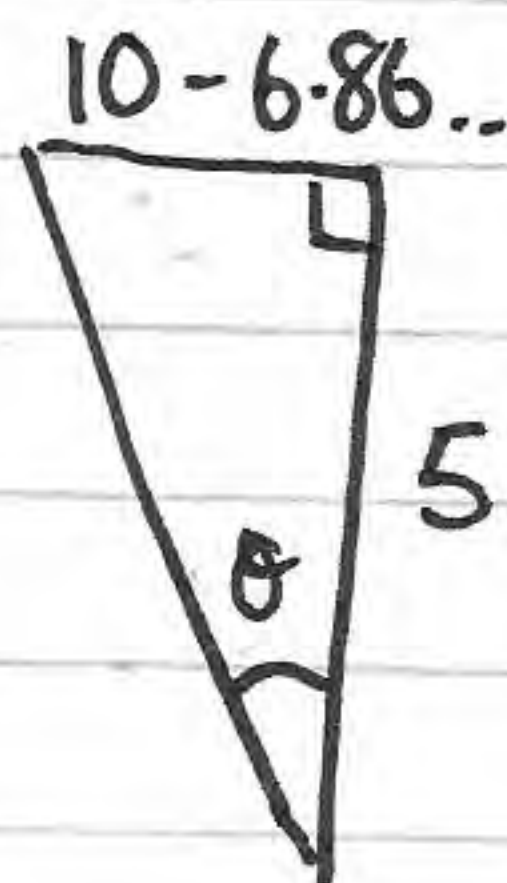
$$M = 100u \quad G(5, 0)$$



$$\frac{25}{2}\pi u g \times \frac{20}{3\pi} + \left(100 - \frac{25}{2}\pi\right)u g \times \bar{x} = 100u g \times 5$$

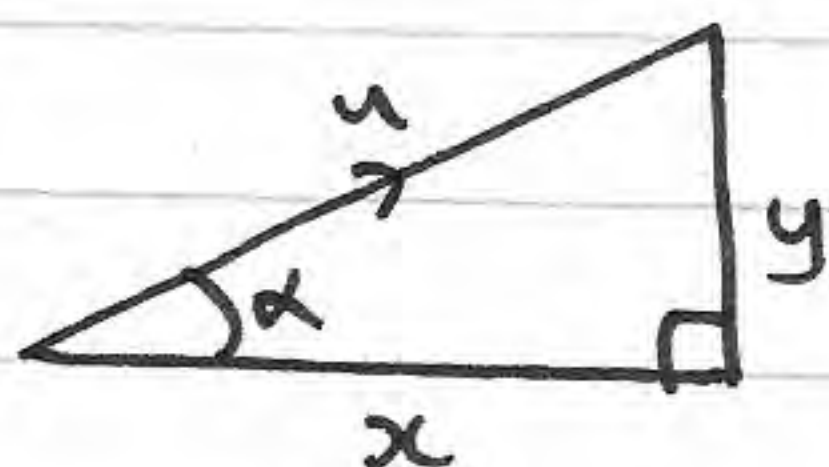
$$\Rightarrow \frac{250}{3} + \left(100 - \frac{25}{2}\pi\right)\bar{x} = 500 \Rightarrow \bar{x} = \underline{6.86 \text{ (3sf)}}$$

b)



$$\theta = \tan^{-1}\left(\frac{10 - 6.86}{5}\right) \Rightarrow \theta = \underline{32.1^\circ}$$

5)



$$\tan \alpha = \frac{y}{x}$$



$$v_{el} = u \cos \alpha \quad x = s$$

$$x = u \cos \alpha \times t$$



$$u = u \sin \alpha$$

$$a = -9.8$$

$$s = y$$

$$\Rightarrow y = u \sin \alpha \times t - 4.9 t^2$$

$$t = \frac{x}{u \cos \alpha}$$

$$\therefore y = \frac{u \sin \alpha \times x}{u \cos \alpha} - \frac{4.9 x^2}{u^2 \cos^2 \alpha}$$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\Rightarrow y = x \tan \alpha - \frac{g x^2}{2 u^2} \times \sec^2 \alpha$$

$$\tan^2 + 1 = \sec^2$$

$$\therefore y = x \tan \alpha - \frac{g x^2}{2 u^2} (\tan^2 \alpha + 1)$$

b) $\alpha = 45 \Rightarrow \tan \alpha = 1$ $-2 = x - \frac{9.8x^2}{2(14^2)} (1^2 + 1)$
 $y = -2$
 $u = 14$

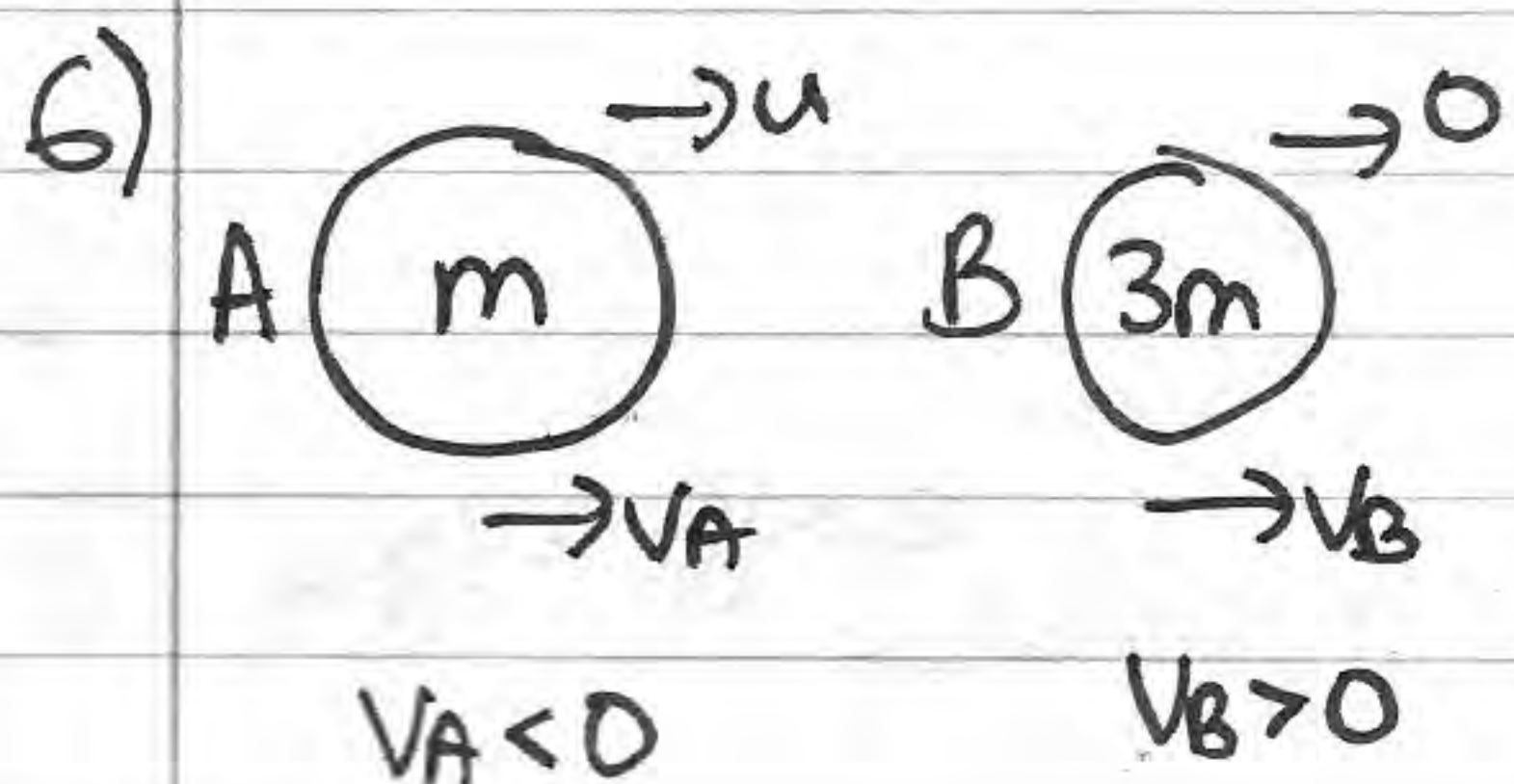
$$\Rightarrow -2 = x - \frac{x^2}{20} \Rightarrow x^2 - 20x - 40 = 0$$

(x20)

$$\Rightarrow x \Rightarrow (x - 10)^2 - 100 = 40$$

$$\Rightarrow x - 10 = \sqrt{140} \Rightarrow x = 10 + \sqrt{140} = \underline{21.8 \text{ m}} \quad (3 \text{ sf})$$

c) $t = \frac{x}{u \cos \alpha} = \frac{21.8 \dots}{14 \cos 45} \Rightarrow t = \underline{2.2 \text{ sec}} \quad (2 \text{ sf})$



$$e = \frac{v_B - v_A}{u} \Rightarrow eu = v_B - v_A$$

$$v_B = eu + v_A$$

$$cuM = m u = m v_A + 3m v_B$$

$$\Rightarrow u = v_A + 3eu + 3v_A$$

$$\Rightarrow 4v_A = u - 3eu$$

$$\Rightarrow v_A = \frac{1}{4}u(1 - 3e)$$

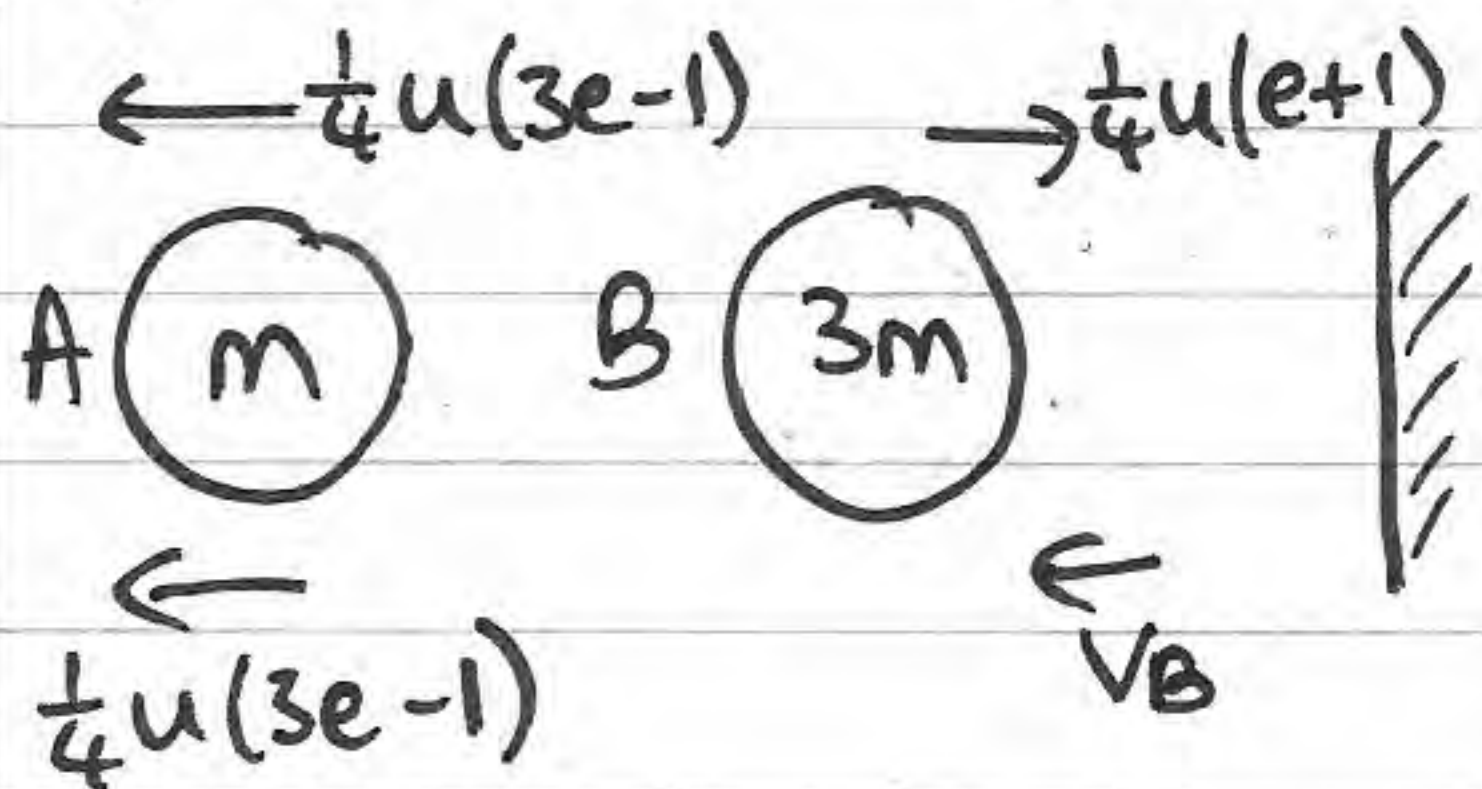
$$\text{Since } v_A < 0 \Rightarrow 3e > 1 \Rightarrow e > \frac{1}{3}$$

$$v_B = eu + \frac{1}{4}u - \frac{3}{4}eu$$

$$\Rightarrow v_B = \frac{1}{4}eu + \frac{1}{4}u \Rightarrow v_B = \frac{1}{4}u(e + 1)$$

Speed A = $\frac{1}{4}u(3e - 1)$

Speed B = $\frac{1}{4}u(e + 1)$



$$e = \frac{4v_B}{\frac{1}{4}u(e + 1)} = \frac{3}{4}$$

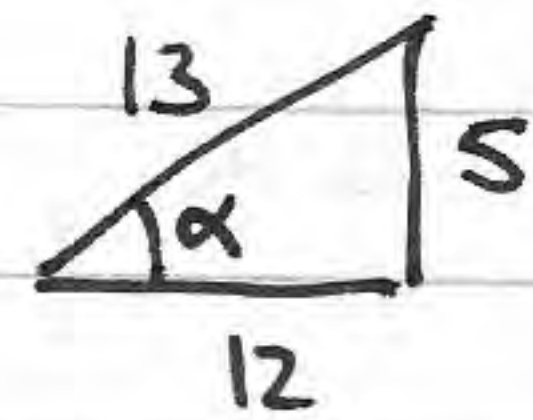
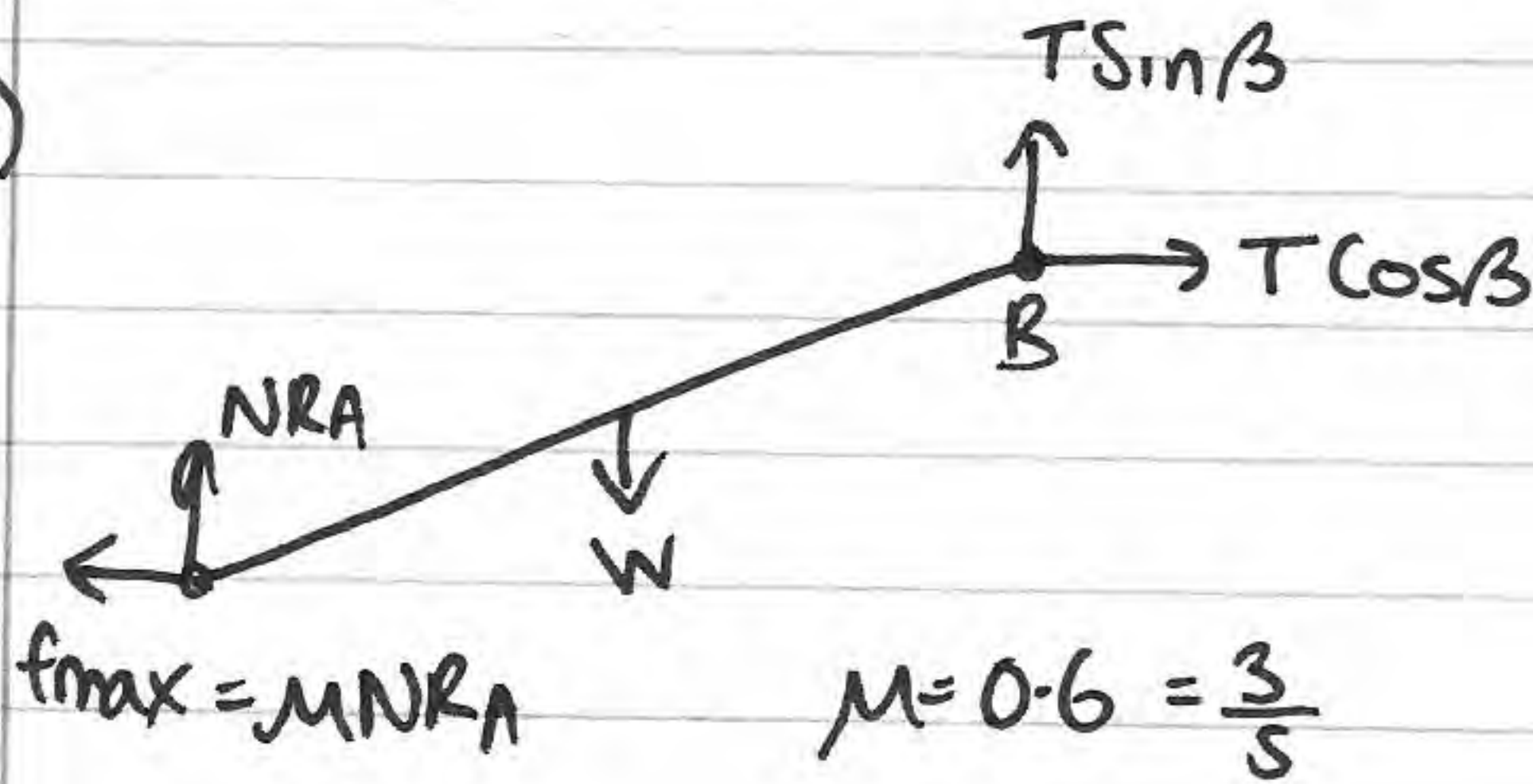
$$v_B = \frac{3u(e + 1)}{16}$$

$$\text{Second collision} \Rightarrow \frac{3\mu(e+1)}{T_6} > \frac{1}{4}\mu(3e-1)$$

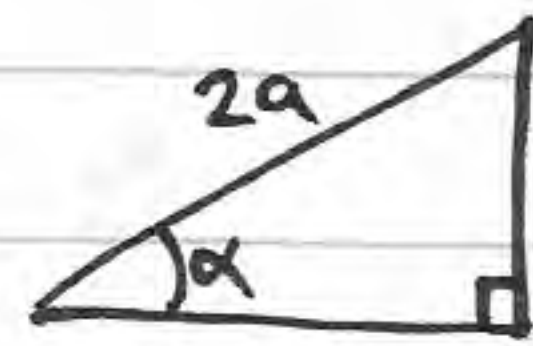
$$\Rightarrow 3e+3 > 12e-4 \Rightarrow 7 > 9e \Rightarrow e < \frac{7}{9}$$

$$\text{from (a) } V_A < 0 \Rightarrow e > \frac{1}{3} \quad \therefore \frac{1}{3} < e < \frac{7}{9}$$

7)



$$\begin{aligned} \tan \alpha &= \frac{5}{12} \\ \sin \alpha &= \frac{5}{13} \\ \cos \alpha &= \frac{12}{13} \end{aligned}$$



$$2a \times \frac{5}{13} = \frac{10}{13}a$$

$$2a \times \frac{12}{13} = \frac{24}{13}a$$

$$W \times a \cos \alpha = N_{RA} \times 2a \sin \alpha + f_{\max} \times 2a \cos \alpha$$

$$\Rightarrow \frac{12}{13}aW = N_{RA} \times \frac{24}{13}a + \frac{3}{5}N_{RA} \times \frac{10}{13}a$$

$$\Rightarrow 12W = 24N_{RA} + 6N_{RA}$$

$$\Rightarrow 12W = 30N_{RA} \quad \therefore N_{RA} = \frac{2}{5}W \quad \#$$

$$\begin{aligned} \text{b) } \sum \vec{F} &= 0 \Rightarrow f_{\max} = T \cos \beta \Rightarrow T \cos \beta = \frac{3}{5}N_{RA} \\ \sum \uparrow &= 0 \Rightarrow T \sin \beta + N_{RA} = W \Rightarrow T \sin \beta = \frac{5}{2}N_{RA} - N_{RA} \end{aligned}$$

$$\therefore \frac{T \sin \beta}{T \cos \beta} = \frac{\frac{5}{2}N_{RA}}{\frac{3}{5}N_{RA}} \Rightarrow \tan \beta = \frac{15}{6} = \frac{5}{2} \Rightarrow \beta = \underline{68.2^\circ}$$

$$\text{c) } T \cos \beta = \frac{3}{5}N_{RA} \Rightarrow T = \frac{\frac{3}{5}N_{RA}}{\cos 68.2^\circ} \Rightarrow T = 1.6155 \dots N_{RA}$$

$$\therefore T = 1.6155 \dots \times \frac{2}{5}W \Rightarrow T = 0.646 \dots W$$

$$\underline{\mu = 0.65 \text{ (2sf)}}$$