**9** A particle is moving in a plane so that its acceleration is $\mathbf{a} = -2\mathbf{j}$. At time $t = 0$ the particle is at the point whose position vector is $2\mathbf{i} - 3\mathbf{j}$ and has velocity $2\mathbf{i} + 4\mathbf{j}$.

a) Find expressions for its velocity and position at time $t$.

b) At what time(s) is the particle moving in the **i**-direction?

c) At what time(s) does the particle cross the $x$-axis?

**10** An aircraft is flying at an altitude of 0.8 km, at a speed of 960 km $h^{-1}$, and on a bearing of 030°. At time $t = 0$ it passes directly over an observer. Taking the observer as the origin and the **i**, **j** and **k**-directions as east, north and upwards respectively:

a) write down the velocity vector, **v**

b) find the position vector, **r**, of the aircraft $t$ hours after passing the observer.

The units for this question are kilometres and hours.

**11** A particle of mass 5 kg is acted upon by a force $(20t\mathbf{i} - 15\mathbf{j})$ N. Initially the particle is at the point with position vector $2\mathbf{i} + 3\mathbf{j}$ and is travelling with velocity $(-2\mathbf{i} + 12\mathbf{j})$ m $s^{-1}$. Find expressions for the velocity and position of the particle at time $t$, and hence find its velocity and position when $t = 6$.

**12** A particle of mass 3 kg is acted upon by a force $(6\mathbf{i} + 3\mathbf{j} + 6(t - 1)\mathbf{k})$ N. Initially the particle is at the origin and is travelling with velocity $(-3\mathbf{j} - 3\mathbf{k})$ m $s^{-1}$ (that is, it is moving perpendicular to the **i**-direction). Show that at one later time it is travelling in the **i**-direction, and find its speed and position at that moment.

**13** The position vector of a particle is given by

$$\mathbf{r} = (2 + 3t + 8t^2)\mathbf{i} + (6 + t + 12t^2)\mathbf{j}$$

a) Find an expression for the velocity of the particle at time $t$.

b) Find an expression for the acceleration of the particle at time $t$.

An observer placed at the origin sees the particle in the direction given by $\tan\theta = \frac{y}{x}$, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ is the position vector of the particle. The direction in which the particle is moving is given by $\tan\phi = \frac{b}{a}$, where $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ is the velocity of the particle. The particle will be moving directly away from or directly towards the observer when $\tan\theta = \tan\phi$.

c) Find the time(s) when the particle is moving directly towards or directly away from the observer

d) Find the position of the particle at the time(s) found in c) and identify whether the particle is moving towards or away from the observer.

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