LET $u = x+y$

$v = \frac{y}{x}$

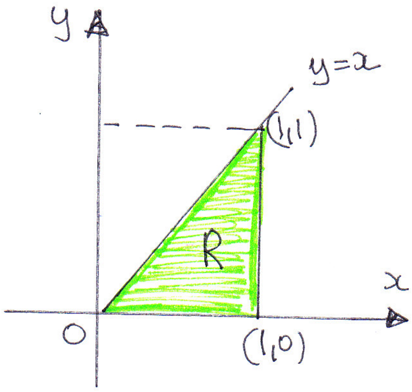
- $\left|\frac{\partial(u,v)}{\partial(x,y)}\right| = \left\|\begin{matrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{matrix}\right\| = \left\|\begin{matrix}1 & 1 \\ -\frac{y}{x^2} & \frac{1}{x}\end{matrix}\right\| = \frac{1}{x} + \frac{y}{x^2} = \frac{x+y}{x^2}$

$du\,dv = \left|\frac{\partial(u,v)}{\partial(x,y)}\right| dx\,dy$

$du\,dv = \frac{x+y}{x^2} dx\,dy$

$$\boxed{dx\,dy = \frac{x^2}{x+y} du\,dv}$$

- NEXT THE INTEGRATION AREA — MIGHT BE HELPFUL TO GET $x = f(u,v)$
$y = g(u,v)$



- $y = vx$

$u = x + vx$

$u = x(1+v)$

$\boxed{x = \frac{v+1}{u}}$

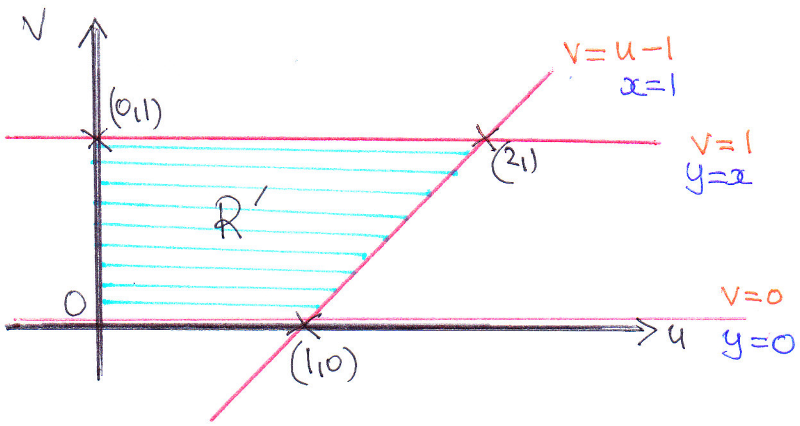
- $y = v\left(\frac{v+1}{u}\right)$

$\boxed{y = \frac{v}{u}(v+1)}$

NOW

- $x = y \Rightarrow \cancel{\frac{v+1}{u}} = \frac{v}{\cancel{u}}\cancel{(v+1)} \Rightarrow \boxed{v=1}$
- $x = 1 \Rightarrow 1 = \frac{v+1}{u} \Rightarrow \boxed{\begin{matrix}u = v+1 \\ \text{OR} \\ v = u-1\end{matrix}}$
- $y = 0 \Rightarrow \boxed{v = 0}$ (FROM ORIGINAL TRANSFORMATIONS)
- ($y = 1$ IS HARD TO TRANSFORM)

DRAW THE INTEGRATION REGION IN THE uv PLANE



FINAL CHECK $(x,y) = \left(\frac{1}{2}, \frac{1}{4}\right)$ LIES IN R

THIS TRANSFORMS TO

$(u,v) = \left(\frac{3}{4}, \frac{1}{2}\right)$

HENCE

$$\iint_R \frac{x+y}{x^2} e^{(x+y)}\, dx\, dy = \iint_{R'} \cancel{\frac{x+y}{x^2}} e^u \left(\cancel{\frac{x^2}{x+y}}\, du\, dv\right)$$

$$= \iint_{R'} e^u\, du\, dv = \int_{v=0}^{1} \int_{u=0}^{u=v+1} e^u\, du\, dv = \int_{v=0}^{1} \left[e^u\right]_{u=0}^{u=v+1} dv$$

$$= \int_0^1 e^{v+1} - 1\, dv = \left[e^{v+1} - v\right]_0^1 = (e^2 - 1) - (e^1 - 0)$$

$$= e^2 - e - 1$$