$$I = \int_0^{\theta} \frac{1}{1 - a\cos x}\,dx$$

$$Let\, t = \tan ½ x \;\; \rightarrow \;\; \frac{dt}{dx} = ½\, sec^2 ½\, x = ½(1+t^2)$$

$$dx = \frac{2}{1+t^2}$$

$$Also\,,\, \cos x = \frac{1-t^2}{1+t^2}$$

$$I = \int_0^{\tan ½\theta} \frac{1}{1 - a\left(\frac{1-t^2}{1+t^2}\right)} \left(\frac{2}{1+t^2}\right) dt$$

$$= 2\int_0^{\tan ½\theta} \frac{1}{1+t^2 - a(1-t^2)}\,dt = 2\int_0^{\tan ½\theta} \frac{1}{1+t^2-a+a\,t^2}\,dt$$

$$= 2\int_0^{\tan ½\theta} \frac{1}{(1+a)t^2 + (1-a)}\,dt$$

Remembering *cough checking* the integral of $\frac{1}{x^2+a^2}$ as $\frac{1}{a}\arctan x$

and using the un-chain rule to account for the coefficient of the variable:

$$I = 2\left(\frac{1}{\sqrt{1-a}\sqrt{1+a}}\right)\left[\arctan\sqrt{\left(\frac{1+a}{1-a}\right)}\,t\right]_0^{\tan ½\theta} = etc\; etc\; don't\; wanna\; type\; it\; out$$

$$\int_0^{\pi/2} \frac{1}{2 - a\cos x}\,dx = ½\int_0^{\pi/2} \frac{1}{1 - \frac{a}{2}\cos x}\,dx$$

$$= ½\left(2\left(\frac{1}{\sqrt{(1-(½ a)^2)}}\right)\right)\left[\arctan\sqrt{\left(\frac{1+½ a}{1-½ a}\right)}\,t\right]_0^{\tan ½(\pi/2)}$$

$$= \frac{1}{\sqrt{(1-a^2/4)}} \arctan\sqrt{\left(\frac{2+a}{2-a}\right)}$$ (The t inside the last bracket = $\tan \pi/4 = 1$)

$$= \frac{2}{\sqrt{(4-a^2)}} \arctan\sqrt{\left(\frac{2+a}{2-a}\right)}$$ (Just multiplying the fraction infront by 2/2)