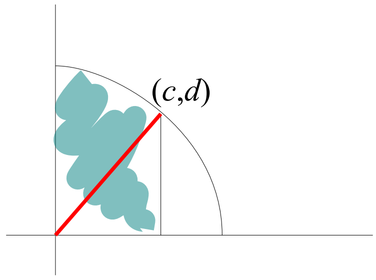
The required area is the part with the blue squiggle in it
Split this area into an arc sector and a triangle
First find $d$ :

$$c^2+d^2=1 \;\rightarrow\; d=(1-c^2)^{½}$$




The area of the triangle (half base times height) is

$$½\,cd=½c\,(1-c^2)^{½}$$

The area of the arc sector is $½\,r^2\theta$
Where $\theta$ is the angle subtended by that arc.
The red line is a radius, so it has length 1, and the line joining $(c\,,d)$
And the y-axis has length $c$.
Arc: $½\times 1^2 \arcsin(c/1)$
Adding these gives what they want

For the next part, we want the corresponding area for an ellipse.
Just integrate:

$$A=\int_0^c b\left(1-\frac{x^2}{a^2}\right)^{½} dx$$

$$=\int_0^c \frac{b}{a}(a^2-x^2)^{½}\,dx$$

Use $x=a\sin t$ $\rightarrow$ New limits: $\arcsin 0$ and $\arcsin(c/a)$ (call this value C to save writing)

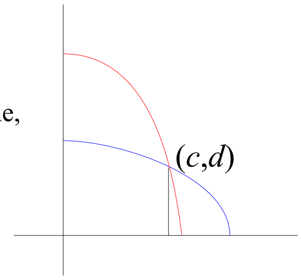
$$dx=a\cos t\,dt$$

$$A=\frac{b}{a}\int_0^C a^2\cos^2 t\,dt=ab\int_0^C ½(1+\cos 2t)\,dt$$

$$=\frac{ab}{2}[t+½\sin 2t]_0^C=\frac{ab}{2}(C+\sin C\cos C)$$

$$\sin C=c/a \text{ and } \cos C=(1-\sin^2 C)^{½}=(1-c^2/a^2)^{½}$$

$$A=\frac{ab}{2}\left(\arcsin\left(\frac{c}{a}\right)+\frac{c}{a}(1-c^2/a^2)^{½}\right)$$




We need the area between two ellipses. $a>b$ means the second one is taller than it is wide.
Integrating the first one from 0 to c, then the second one from c to b gives the required area.
c is the x co-ordinate of the intersection. So we need the area under the blue line up till the black line, and then the area under the red line.

Finding c: $\frac{c^2}{a^2}+\frac{d^2}{b^2}=1$ and $\frac{c^2}{b^2}+\frac{d^2}{a^2}=1$ Multiplying the first by $a^2b^4$ and the second by $a^4b^2$:

$$b^4c^2+a^2b^2d^2=a^2b^4 \quad\text{and}\quad a^4c^2+a^2b^2d^2=a^4b^2$$

Subtracting: $(a^4-b^4)c^2=a^2b^2(a^2-b^2)$ $\rightarrow$ $c^2=\frac{a^2b^2}{(a^2+b^2)}$ (difference of squares on LHS)

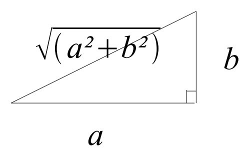
We already have the area under the blue: $\frac{ab}{2}\left(\arcsin\left(\frac{c}{a}\right)+\left(\frac{c}{a}\right)(1-c^2/a^2)^{½}\right)$

$$=\frac{ab}{2}\left(\arcsin\left(\frac{b}{\sqrt{a^2+b^2}}\right)+\frac{ab}{a^2+b^2}\right)*$$

The area under the red is found by integration, but it works out largely the same,
except a and b are switched, and the limits are different:

$$\left(\frac{ab}{2}\right)[t+\sin t\cos t]_{\arcsin(c/b)}^{\arcsin(b/b)}=\left(\frac{ab}{2}\right)\left(\pi/2-\arcsin(c/b)-\frac{c}{b}\left(1-\frac{c^2}{b^2}\right)^{½}\right)=\left(\frac{ab}{2}\right)\left(\pi/2-\arcsin\left(\frac{a}{\sqrt{(a^2+b^2)}}\right)-\frac{a}{\sqrt{(a^2+b^2)}}\left(1-\frac{a^2}{a^2+b^2}\right)^{½}\right)$$

$$=\left(\frac{ab}{2}\right)\left(\pi/2-\arcsin\left(\frac{a}{\sqrt{(a^2+b^2)}}\right)-\frac{ab}{a^2+b^2}\right)\text{after a bit of algebra .}$$




considering the triangle on the right shows that $\pi/2-\arcsin\left(\frac{a}{\sqrt{(a^2+b^2)}}\right)=\arcsin\left(\frac{b}{\sqrt{(a^2+b^2)}}\right)$

So this area is: $\frac{ab}{2}\left(\arcsin\left(\frac{b}{\sqrt{a^2+b^2}}\right)-\frac{ab}{a^2+b^2}\right)$
Adding it to the other area (*) and multiplying by 4, since it happens in all four quadrants, gives the required answer.