Pascal's triangle goes 1,4,6,4,1 for the power of 4 row:

$$(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$$
$$(x-1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$$

The given equation becomes:

$$2x^4 + 12x^2 + 2 - c = 0$$

This is a quadratic in x^2:

$$x^2 = (\frac{1}{4})(-12 \pm \sqrt{(12^2 - 4(2)(2-c))})$$

For x to be real, $x^2 \geqslant 0$. So we can ignore the negative solution of this.
For the root with the positive square root to be non-negative:

$$-12 + \sqrt{(12^2 - 4(2)(2-c))} \geqslant 0$$

$\sqrt{(144 - 8(2-c))} \geqslant 12$ Both sides are positive, so squaring is allowed:

$$144 - 8(2-c) \geqslant 144$$

$0 \geqslant 2 - c \rightarrow c \geqslant 2$ For real roots.

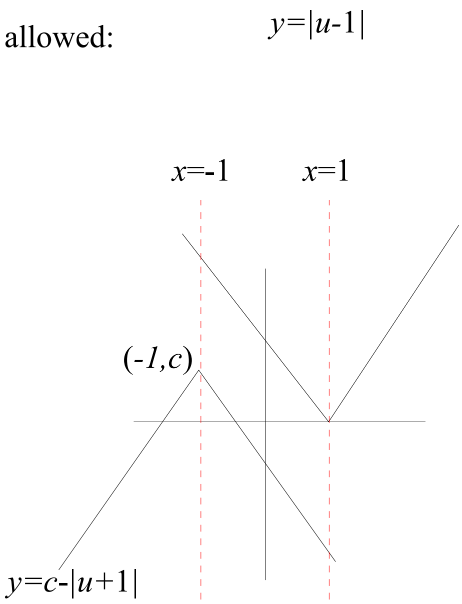
Now if $c=0$ then $x^2=0$ so x has only one value.

If we consider $u = x - 2$ part (ii) becomes part (i)
So the same conditions apply.

Using $u = x - 2$ again, we can write part (iii) as:

$$|u-1| = c - |u+1|$$

A sketch is given:



The vertex of the unhappy graph is at $x=-1$, so if $c=2$ then part of the two graps lie ontop of each other.
This means infinite solutions?
If the vertex is above that point, then there are two solutions.
There are none otherwise.

Using the same strategy, part (iv) becomes:

$$(u-1)^3 + (u+1)^3 = c$$
$$2u^3 + 6u - c = 0$$

This is a cubic, so it either has one or three real roots.
Differentiating reveals:

$$\frac{dy}{du} = 6u^2 + 6 \neq 0$$

No turning points is a sufficient condition for a cubic to have only one root.