Extension Exercise

Numerical answers should be calculated without a calculator. A calculator may, however, be used for checking purposes.

1 **a** Find the first three terms in the expansion, in ascending powers of x, of $(1-x)^4$.

b Repeat part **a** for $(1+3x)^5$.

c Hence find the coefficient of x^2 in the expansion of $(1-x)^4(1+3x)^5$.

2 Expand

a $(2+x+x^2)^3$ **b** $(3+x-2x^2)^3$ **c** $(1-x+x^2)^4$

3 Expand as far as the term in x^2

a $(1-x-x^2)^4$ **b** $(1+2x+3x^2)^5$ **c** $(2+x-x^2)^6$

4 Use a binomial expansion to simplify these.

a $(\sqrt{3}+\sqrt{2})^5+(\sqrt{3}-\sqrt{2})^5$ **b** $(2\sqrt{2}+1)^4-(2\sqrt{2}-1)^4$

5 **a** Expand $(1+2x)^3$.

b Hence, find the term in x^2 in the expansion of $(2-x+x^2)(1+2x)^3$.

6 Use a binomial expansion to evaluate 1.01^{10}, correct to 5 significant figures.

7 **a** Find the first three terms in the expansions, in ascending powers of x, of

i $(1+3x)^6$ **ii** $(1-x)^6$

b Hence find the coefficient of x^2 in the expansion of $(1+2x-3x^2)^6$.

8 Expand $(x+2)^5$ and $(x-2)^4$. Obtain the coefficient of x^7 in the product of the expansions.

9 **a** Find the first four terms of the expansion of $(1-x)^5$.

b By expressing $(1-x+2x^2)^5$ as $(1-(x-2x^2))^5$, find, as far as the term in x^3, the expansion of $(1-x+2x^2)^5$.

10 **a** Find the middle term of the expansion of $(2x+3)^8$ and the value of this term when $x=\frac{1}{12}$.

b Find the constant term in the expansion of $\left(x^2+\dfrac{2}{x}\right)^9$

11 Find the first four terms in the expansion of $(1-x+2x^2)^5$ in ascending powers of x.

12 **a** Use the expansion of $(1+x)^n$ to show that, for $n \in \mathbb{Z}^+$

i $\dbinom{n}{0}+\dbinom{n}{1}+\dbinom{n}{2}+\dbinom{n}{3}+\cdots+\dbinom{n}{n}=2^n$

ii $\dbinom{n}{0}-\dbinom{n}{1}+\dbinom{n}{2}-\dbinom{n}{3}+\cdots+\dbinom{n}{n}=0$

b Interpret the results in part **a** in the context of Pascal's triangle.

13 **a** Prove that

i $\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$ **ii** $\binom{n+2}{3} - \binom{n}{3} = n^2$

b Interpret the results in part **a** in the context of Pascal's triangle.

14 Prove, from first principles, that, for $n \in \mathbb{Z}^+$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

