

Question 15

The surface S is the sphere with Cartesian equation

$$x^2 + y^2 + z^2 = 4$$

- a) By using Spherical Polar coordinates, (r, θ, ϕ) , evaluate by direct integration the following surface integral

$$I = \iint_S (x^4 + xy^2 + z) dS.$$

- b) Verify the answer of part (a) by using the Divergence Theorem.

256π
5

The image displays two pages of handwritten mathematical work. The left page, titled 'a) $\int_S x^4 + xy^2 + z \, dS = \dots$ SWITCH INTO SPHERICAL COORDS', shows the conversion of the surface integral into spherical coordinates. It includes a diagram of a sphere with radius 2, and the integral is evaluated using the volume element $dV = r^2 \sin \theta \, dr \, d\theta \, d\phi$. The right page, titled 'b) Verify the answer of part (a) by using the Divergence Theorem', shows the application of the Divergence Theorem. It calculates the divergence of the vector field $\mathbf{F} = (x^4, xy^2, z)$ and integrates it over the volume of the sphere. Both pages include detailed annotations and diagrams.