Vertical Motion - Get time of flight

$s = ut + \frac{1}{2}at^2$

$s = Vtsin\alpha - \frac{1}{2}gt^2$
$\frac{1}{2}gt^2 - Vtsin\alpha + s = 0$
$t = \frac{Vsin\alpha + \sqrt{V^2sin^2\alpha - 4.\frac{1}{2}g.s}}{g}$
$t = \frac{Vsin\alpha + \sqrt{V^2sin^2\alpha - 2gs}}{g}$

Horizontal Motion

$S_{\mathrm{H}} = V_{\mathrm{H}} * t$
$S_{\mathrm{H}} = \frac{Vcos\alpha}{g}(Vsin\alpha + \sqrt{V^2sin^2\alpha - 2gs}$
$\frac{dS_{\mathrm{H}}}{d\alpha} = -\frac{Vsin\alpha}{g}(Vsin\alpha + \sqrt{V^2sin^2\alpha - 2gs}) + \frac{Vcos\alpha}{g}(Vcos\alpha + \frac{1}{2}.2V^2sin\alpha.cos\alpha(V^2sin^2\alpha - 2gs)^{-\frac{1}{2}}) = 0$
$sin\alpha(Vsin\alpha + \sqrt{V^2sin^2\alpha - 2gs}) = cos\alpha(Vcos\alpha + V^2sin\alpha.cos\alpha(V^2sin^2\alpha - 2gs)^{-\frac{1}{2}})$

Multiplying both sides by $\sqrt{V^2sin^2\alpha - 2gs}$

$Vsin\alpha.\sqrt{V^2sin^2\alpha - 2gs} + V^2sin^2\alpha - 2gs = \frac{Vcos^2\alpha}{sin\alpha}.\sqrt{V^2sin^2\alpha - 2gs} + V^2cos^2\alpha$
$\sqrt{V^2sin^2\alpha - 2gs}(Vsin\alpha - \frac{Vcos^2\alpha}{sin\alpha}) = V^2(cos^2\alpha - sin^2\alpha) + 2gs$
$\sqrt{V^2sin^2\alpha - 2gs}(\frac{Vsin^2\alpha - Vcos^2\alpha)}{sin\alpha}) = V^2(cos^2\alpha - sin^2\alpha) + 2gs$
$V.\sqrt{V^2sin^2\alpha - 2gs}(sin^2\alpha - cos^2\alpha) = V^2sin\alpha(cos^2\alpha - sin^2\alpha) + 2gs.sin\alpha$

now squaring both sides ...

$V^2(V^2sin^2\alpha - 2gs)(sin^2\alpha - cos^2\alpha)^2 = V^4sin^2\alpha(cos^2\alpha - sin^2\alpha)^2 + 4g^2s^2sin^2\alpha + 4gsV^2sin^2\alpha(cos^2\alpha - sin^2\alpha)$

let $u = sin^2\alpha$

$V^2(V^2u - 2gs)(2u-1)^2 = V^4u(2u-1)^2 + 4g^2s^2u + 4gsV^2u(1-2u)$
$V^4u(2u-1)^2 - 2gsV^2(2u-1)^2 = V^4u(2u-1)^2 + 4g^2s^2u + 4gsV^2u(1-2u)$

cancelling the term, $V^4u(2u-1)^2$, from both sides,

$-2gsV^2(2u-1)^2 = 4g^2s^2u + 4gsV^2u(1-2u)$

$-V^2(2u-1)^2 = 2gsu + 2uV^2(1-2u)$
$-V^2(2u-1)^2 + 2uV^2(2u-1) = 2gsu$
$-V^2(2u-1)\{(2u-1) - 2u\} = 2gsu$
$V^2(2u-1) = 2gsu$
$2V^2u - V^2 = 2gsu$
$u(2V^2 - 2gs) = V^2$
$u = \frac{V^2}{2(V^2-gs)}$
$\underline{\underline{sin^2\alpha = \frac{V^2}{2(V^2-gs)}}}$

Where,
α is the angle of trajectory,
V is the speed of trajectory,
s is the distance below the cliff.

N.B. s is measured +ve upwards, so s should be entered as a negative value in the above formula. e.g. s = -100.