QUESTION PART REFERENCE

Core 4 June 2015

1(a) Express $f(x) = \frac{(19x-2)}{(5-x)(1+6x)}$ in the form $f(x) = \frac{A}{5-x} + \frac{B}{1+6x}$

$19x-2 = A(1+6x) + B(5-x)$

Let $x=5$ $\therefore 19(5)-2 = A(1+6(5)) + B(0)$

$93 = 31A$

$\therefore A = 3$

Let $x = \frac{-1}{6}$ $\therefore 19\left(\frac{-1}{6}\right)-2 = A(0) + B\left(5-\left(\frac{-1}{6}\right)\right)$

$-\frac{31}{6} = \frac{31}{6}B$

$\therefore B = -1$

$\therefore f(x) = \frac{3}{5-x} - \frac{1}{1+6x}$

(b) Find $\int_0^4 f(x)\,dx$. Show that it equals $p\ln 5$ where p is rational.

$$\int_0^4 \frac{3}{5-x} - \frac{1}{1+6x}\,dx = \left[-3\ln|5-x| - \tfrac{1}{6}\ln|1+6x|\right]_0^4$$

$$\underbrace{\left(-3\ln 1 - \tfrac{1}{6}\ln 25\right)}_{=0} - \underbrace{\left(-3\ln 5 - \tfrac{1}{6}\ln 1\right)}_{=0}$$

$\therefore -\frac{1}{6}\ln 25 + 3\ln 5$

$\Rightarrow -\frac{1}{6}\ln 5^2 + 3\ln 5 \Rightarrow 2\left(\frac{-1}{6}\right)\ln 5 + 3\ln 5$

$\therefore \frac{8}{3}\ln 5$