5. A curve is defined by parametric equations $x = \cos 2t$, $y = \sin t$
At the point P, $t = \frac{\pi}{6}$

(a) Find the gradient at P.

$$x = \cos 2t \qquad y = \sin t$$

$$\therefore \frac{dx}{dt} = -2\sin 2t \qquad \therefore \frac{dy}{dt} = \cos t$$

$$\therefore \frac{dy}{dx} = \frac{dt}{dx} \times \frac{dy}{dt} = \frac{\cos t}{-2\sin 2t}$$

$$\text{At } t = \frac{\pi}{6},\ \frac{dy}{dx} = \frac{\cos(\pi/6)}{-2\sin(\frac{2\pi}{6})} = \underline{\underline{\frac{-1}{2}}}$$

(b) Find the equation of the normal to the curve at P in the form $y = mx + c$

At P gradient $= \frac{-1}{2}$ $\therefore$ gradient of normal $= 2$

$$y - y_1 = m(x - x_1) \qquad x = \cos\left(\frac{2\pi}{6}\right) = \frac{1}{2},\ y = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$y - \frac{1}{2} = 2\left(x - \frac{1}{2}\right) \quad \therefore \underline{\underline{y = 2x - \frac{1}{2}}}$$

QUESTION PART REFERENCE

(c) The normal at P cuts the curve at Q$(\cos 2q, \sin q)$. Set up a quadratic equation in $\sin q$ to find where the normal cuts the curve again and find the x-coordinate of Q.

$y = 2x - \frac{1}{2}$ $\therefore$ $\sin q = 2\cos 2q - \frac{1}{2}$

$\boxed{\cos 2q = \cos^2 q - \sin^2 q}$ $\boxed{\sin^2 q + \cos^2 q = 1}$

$\therefore 2\cos 2q = 2((1-\sin^2 q) - \sin^2 q)$

$= 2 - 4\sin^2 q$

$\therefore 4\sin^2 q + \sin q - \frac{3}{2} = 0$

$(\times 2)$ $8\sin^2 q + 2\sin q - 3 = 0$

$(2\sin q - 1)(4\sin q + 3) = 0$

$\therefore \sin q = \frac{1}{2}, -\frac{3}{4}$

$\therefore q = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ AND $q = \sin^{-1}\left(\frac{-3}{4}\right) = -0.848062079$

$\therefore \cos 2q$ is the x-coordinate at Q $= \cos(2(-0.848\ldots))$

$\therefore$ x-coordinate at Q $= -\frac{1}{8}$