QUESTION PART REFERENCE

6.(a) The points A and B have coordinates (3,2,10) and (5,-2,4) respectively.
The line l has equation $\underline{r} = \begin{pmatrix}3\\2\\10\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}$

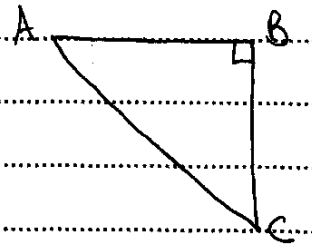
Find the angle between the line l and $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix}5\\-2\\4\end{pmatrix} - \begin{pmatrix}3\\2\\10\end{pmatrix} = \begin{pmatrix}2\\-4\\-6\end{pmatrix}$$

$$\cos\theta = \frac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|} \Rightarrow \frac{\begin{pmatrix}2\\-4\\-6\end{pmatrix}\cdot\begin{pmatrix}3\\1\\-2\end{pmatrix}}{\sqrt{2^2+4^2+6^2}\times\sqrt{3^2+1^2+2^2}} = \frac{14}{28} = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\tfrac{1}{2}\right) = 60°$$

(b) The point C lies on the line l such that the angle ABC is 90°.
Find the coordinates of the point C.



$$\overrightarrow{AB}\cdot\overrightarrow{CB} = 0$$

$$\overrightarrow{CB} = \underline{b} - \underline{c} = \begin{pmatrix}5\\-2\\4\end{pmatrix} - \begin{pmatrix}3+3\lambda\\2+\lambda\\10-2\lambda\end{pmatrix}$$

$$\overrightarrow{CB} = \begin{pmatrix}2-3\lambda\\-4-\lambda\\-6+2\lambda\end{pmatrix}$$

$$\overrightarrow{AB}\cdot\overrightarrow{CB} = \begin{pmatrix}2\\-4\\-6\end{pmatrix}\cdot\begin{pmatrix}2-3\lambda\\-4-\lambda\\-6+2\lambda\end{pmatrix} = 0$$

$$\therefore 2(2-3\lambda) - 4(-4-\lambda) - 6(-6+2\lambda) = 0$$

$$4 - 6\lambda + 16 + 4\lambda + 36 - 12\lambda = 0$$

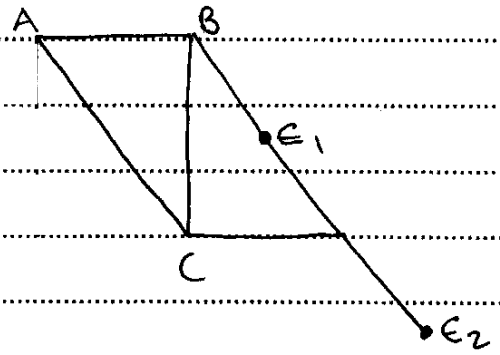
$$56 - 14\lambda = 0$$

$$\therefore \lambda = 4 \quad \therefore C = (15, 6, 2)$$

QUESTION PART REFERENCE

Answer space for question 4

(c) The point D is such that BD is parallel to AC. The point E is such that DE is half that of BD. Find the coordinates of the 2 possible points of E



$\overrightarrow{OE} = \overrightarrow{OB} + \frac{1}{2}\overrightarrow{AC}$ $\quad \overrightarrow{AC} = \underline{c} - \underline{a} = \begin{pmatrix}15\\6\\2\end{pmatrix} - \begin{pmatrix}3\\2\\10\end{pmatrix} = \begin{pmatrix}12\\4\\-8\end{pmatrix}$

$\Rightarrow \begin{pmatrix}5\\-2\\4\end{pmatrix} + \frac{1}{2}\begin{pmatrix}12\\4\\-8\end{pmatrix}$

$= \begin{pmatrix}11\\0\\0\end{pmatrix}$ $\therefore E_1 = (11, 0, 0)$

OR $\quad \overrightarrow{OE} = \overrightarrow{OB} + \frac{3}{2}\overrightarrow{AC}$

$\Rightarrow \begin{pmatrix}5\\-2\\4\end{pmatrix} + \frac{3}{2}\begin{pmatrix}12\\4\\-8\end{pmatrix}$

$= \begin{pmatrix}23\\4\\-8\end{pmatrix}$ $\therefore E_2 = (23, 4, -8)$