QUESTION PART REFERENCE

Answer space for question 4

8. (a) Initially a pond is empty, the depth of the pond is x at time t. The pond is filled with water according to the differential equation $\frac{dx}{dt} = \frac{\sqrt{4+5x}}{5(1+t)^2}$

Find x as a function of t.

$$\int \frac{5}{\sqrt{4+5x}}\,dx = \int \frac{1}{(1+t)^2}\,dt$$

$$\therefore \int 5(4+5x)^{-1/2}\,dx = \int (1+t)^{-2}\,dt$$

$$\Rightarrow 2(4+5x)^{1/2} = -1(1+t)^{-1} + c$$

At $t=0$, $x=0$

$$\therefore 2(4+5(0))^{1/2} = -1(1+(0))^{-1} + c$$

$$\therefore 4+1 = c$$

$$c = 5$$

$$\therefore 2(4+5x)^{1/2} = -1(1+t)^{-1} + 5$$

$(\div 2)$

$$\therefore (4+5x)^{1/2} = -\frac{1}{2}(1+t)^{-1} + \frac{5}{2}$$

square both sides $\therefore 4+5x = \left(-\frac{1}{2}(1+t)^{-1} + \frac{5}{2}\right)^2$

(-4) $\therefore 5x = \left(-\frac{1}{2}(1+t)^{-1} + \frac{5}{2}\right)^2 - 4$

$(\div 5)$ $\therefore x = \frac{1}{5}\left[\left(-\frac{1}{2}(1+t)^{-1} + \frac{5}{2}\right)^2 - 4\right]$

QUESTION PART REFERENCE

8 (b)(i) Another pond models a circle of radius r. The rate of change of radius r is inversely proportional to the pond surface area. Write a differential equation to model this situation

$$\frac{dr}{dt} \propto \frac{1}{A} \quad \therefore \frac{dr}{dt} = \frac{k}{\pi r^2}$$

(ii) The rate of change of radius r is 4.5 when the radius is 1 metre. Find the radius when the rate is 0.5

$$\frac{dr}{dt} = \frac{k}{\pi r^2} \quad \therefore 4.5 = \frac{k}{\pi (1)^2} \quad \therefore k = 4.5\pi$$

$$\text{At } \frac{dr}{dt} = 0.5 \Rightarrow 0.5 = \frac{4.5\cancel{\pi}}{\cancel{\pi} r^2} = \frac{4.5}{r^2}$$

$$\therefore r^2 = \frac{4.5}{0.5} = 9$$

$$\therefore r = \pm 3$$

radius must be 3 metres as r must be positive

$$r = 3m$$