Question

Given the trigonometric equation

$$\frac{\sin(x-\alpha)}{\cos(x-\alpha)-2\tan\alpha\sin(x-\alpha)}=\tan\alpha\,,$$

show clearly that

$$\tan x = 2\tan\alpha\,.$$

Question

The point P whose x coordinate is 1 lies on a curve with equation $y=f(x)$.

The tangent to the curve at P passes through the point $Q(2,2)$.

Given that the tangent to the curve at the point where $x=0$ has gradient 2, determine the equation of $f(x)$.

Question

By using the substitution $x=2\tan^2\theta$, or otherwise, find

$$\int \frac{2-x}{\sqrt{x}(x+2)^2}\,dx\,.$$

Question

Determine the two real roots of the equation

$$(x-7)(x-3)(x+5)(x+1)=1680\,.$$

Question

With respect to a fixed origin O, the points A, B and C have position vectors

$$\mathbf{a} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 8 \\ 2 \\ 7 \end{pmatrix} \text{ and } \mathbf{c} = \begin{pmatrix} 11 \\ 0 \\ 1 \end{pmatrix}.$$

a) Determine the volume of the cube.

The points P, Q and R are vertices of a different cube, so that

$$\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 1 \\ 7 \end{pmatrix} \text{ and } \overrightarrow{PR} = \begin{pmatrix} k \\ 4 \\ 3 \end{pmatrix},$$

where k is a positive constant.

b) Given that $\measuredangle QPR = 60°$, determine …

i. … the value of k.

ii. … the length of the diagonal of the second cube.

Question

A sequence u_1, u_2, u_3, u_4, u_5 … is given by the recurrence formula

$$u_{n+2} = \frac{3u_n + u_{n+1}}{2}, \quad u_1 = 1, \ u_2 = 1.$$

It is further given that in this sequence **the ratio of consecutive terms** converges to a limit L.

Determine the value of L.

Question

The straight line with equation

$$y = t(x-2),$$

where t is a parameter, crosses the circle with equation

$$x^2 + y^2 = 1$$

at two distinct points A and B.

a) Show that the coordinates of the midpoint of AB are given by

$$M\left(\frac{2t^2}{1+t^2}, -\frac{2t}{1+t^2}\right).$$

b) Hence show that the locus of M as t varies is a circle, stating its radius and the coordinates of its centre.

Question

Express

$$\frac{1}{\sqrt{5+\sqrt{24}}},$$

in the form $\sqrt{p} - \sqrt{q}$, where p and q are integers.

Question

The k^{th} of an arithmetic progression is 849, where k is a positive integer.

The $(k+p)^{\text{th}}$ term and the $(k+2p+1)^{\text{th}}$ term of the same arithmetic progression are 873 and 905 respectively, where p is a positive integer.

Find the value of the $(k+20)^{\text{th}}$ term of the progression.