Paper Reference(s)

6676
Edexcel GCE
Pure Mathematics P6

Advanced/Advanced Subsidiary
Thursday 20 June 2002 – Morning
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Pure Mathematics P6), the paper reference (6676), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Prove that $\sinh(\mathrm{i}\pi - \theta) = \sinh\theta$.

(4)

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2. The variable y satisfies the differential equation

$$\frac{dy}{dx} = x + e^{y}.$$

It is given that $y = 1$ at $x = 0.5$. Use the approximation $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$, with $h = 0.1$, to find an estimate of y to 4 decimal places at

(*a*) $x = 0.6$,

(3)

(*b*) $x = 0.7$.

(2)

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3. (*a*) The point P represents a complex number z in an Argand diagram. Given that

$$|z - 2\mathrm{i}| = 2|z + \mathrm{i}|,$$

(i) find a cartesian equation for the locus of P, simplifying your answer.

(2)

(ii) sketch the locus of P.

(3)

(*b*) A transformation T from the z-plane to the w-plane is a translation $-7 + 11\mathrm{i}$ followed by an enlargement with cente the origin and scale factor 3.

Write down the transformation T in the form

$$w = az + b, \quad a, b \in \mathbb{C}.$$

(2)

4.

$$y\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + y = 0.$$

(*a*) Find an expression for $\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}$.

(5)

Given that $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ at $x = 0$,

(*b*) find the series solution for y, in ascending powers of x, up to an including the term in x^3.

(5)

(*c*) Comment on whether it would be sensible to use your series solution to give estimates for y at $x = 0.2$ and at $x = 50$.

(2)

5.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 4 \\ 0 & 5 & 4 \\ 4 & 4 & 3 \end{pmatrix}.$$

(*a*) Verify that $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of **A** and find the corresponding eigenvalue.

(3)

(*b*) Show that 9 is another eigenvalue of **A** and find the corresponding eigenvector.

(5)

(*c*) Given that the third eigenvector of **A** is $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, write down a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{A}\mathbf{P} = \mathbf{D}.$$

(5)

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6. For $n \in \mathbb{Z}^+$ prove that

(a) $2^{3n+2} + 5^{n+1}$ is divisible by 3,

(9)

(b) $\begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & -n \\ 9n & 3n+1 \end{pmatrix}$.

(7)

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7. The plane Π passes through the points

$$A(-1, -1, 1),\ B(4, 2, 1) \text{ and } C\ (2, 1, 0).$$

(a) Find a vector equation of the line perpendicular to Π which passes through the point D (1, 2, 3).

(3)

(b) Find the volume of the tetrahedron $ABCD$.

(3)

(c) Obtain the equation of Π in the form $\mathbf{r.n} = p$.

(3)

The perpendicular from D to the plane Π meets Π at the point E.

(d) Find the coordinates of E.

(4)

(e) Show that $DE = \dfrac{11\sqrt{35}}{35}$.

(2)

The point D' is the reflection of D in Π.

(f) Find the coordinates of D'.

(3)

END