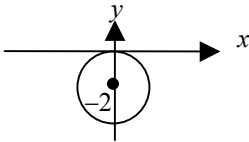


EDEXCEL PURE MATHEMATICS P6 (6676) – JUNE 2002 PROVISIONAL MARK SCHEME

| Question Number | Scheme                                                                                                                                                                                                                                                                                                                                                                                                          | Marks                                                                        |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| 1.              | $\sinh(i\pi - \theta) = \frac{e^{i\pi - \theta} - e^{-(i\pi - \theta)}}{2}$ $= \frac{(-1)e^{-\theta} - e^{\theta} \div (-1)}{2}$ $= \frac{-e^{-\theta} + e^{\theta}}{2}$ $= \sinh \theta \quad (\text{fml}) \quad \text{cso}$                                                                                                                                                                                   | <p>M1</p> <p>B1</p> <p>A1</p> <p>A1</p>                                      |
| Alt.            | $\sinh(i\pi - \theta) = \sinh i\pi \cosh \theta - \cosh i\pi \sinh \theta$ $= 0 - (-1) \sinh \theta$ $= \sinh \theta \quad (\text{fml}) \quad \text{cso}$                                                                                                                                                                                                                                                       | <p>M1</p> <p>B1, A1</p> <p>A1</p> <p><b>(4 marks)</b></p>                    |
| 2.              | $x_0 = 0.5, \quad y_0 = 1 \quad \left(\frac{dy}{dx}\right)_0 = 0.5 + e = 3.2183$ $y_1 = y_0 + h\left(\frac{dy}{dx}\right)_0 = 1 + 0.1(0.5 + e) = 1.05 + 0.1e$ $= 1.3218$ <p>Now <math>x_1 = 0.6, \quad y_1 = 1.3218 \quad \therefore \left(\frac{dy}{dx}\right)_1 = 0.6 + e^{1.3218} = 4.3502</math></p> <p>and <math>y_2 = 1.3218 + 0.1(4.3502) = 1.7568 \quad (\text{or } 1.7569) \quad \text{cao}</math></p> | <p>B1</p> <p>M1</p> <p>A1 ft</p> <p>M1</p> <p>A1</p> <p><b>(5 marks)</b></p> |
| 3. (a)(i)       | $ x + (y - 2)i  = 2 x + (y + i) $ $\therefore x^2 + (y - 2)^2 = 4(x^2 + (y + 1)^2)$                                                                                                                                                                                                                                                                                                                             | M1                                                                           |
| (ii)            | <p>so <math>3x^2 + 3y^2 + 12y = 0</math> any correct from; 3 terms; isw</p> <div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <p>Sketch circle</p> <p>Centre (0, -2)</p> <p><math>r = 2</math> or touches axis</p> </div> </div>                                                           | <p>A1 <b>(2)</b></p> <p>B1</p> <p>B1</p> <p>B1 <b>(3)</b></p>                |
| (b)             | $w = 3(z - 7 + 11i)$ $= 3z - 21 + 33i$                                                                                                                                                                                                                                                                                                                                                                          | <p>B1</p> <p>B1 <b>(2)</b></p> <p><b>(7 marks)</b></p>                       |

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| Question Number | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Marks                                                                                                               |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| 4.              | <p>(a) <math>y \frac{d^3 y}{dx^3} + \frac{dy}{dx} \frac{d^2 y}{dx^2}; + 2 \left( \frac{dy}{dx} \right) \frac{d^2 y}{dx^2}; + \frac{dy}{dx} = 0</math> marks can be awarded in (b)</p> <p><math>\frac{d^3 y}{dx^3} = \frac{-3 \frac{dy}{dx} \frac{d^2 y}{dx^2} - \frac{dy}{dx}}{y}</math> or sensible correct alternative</p> <p>(b) When <math>x = 0</math> <math>\frac{d^2 y}{dx^2} = -2</math>, and <math>\frac{d^3 y}{dx^3} = 5</math></p> <p><math>\therefore y = 1 + x - x^2 + \frac{5}{6}x^3 \dots</math></p> <p>(c) Could use for <math>x = 0.2</math> but not for <math>x = 50</math> as approximation is best at values close to <math>x = 0</math></p> | <p>M1 A1; B1;B1</p> <p>B1 (5)</p> <p>M1A1, A1 ft</p> <p>M1, A1 ft (5)</p> <p>B1</p> <p>B1 (2)</p> <p>(12 marks)</p> |

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| Question Number | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Marks                                                                                                                     |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------|
| 5.              | <p>(a) <math>\begin{pmatrix} 1 &amp; 0 &amp; 4 \\ 0 &amp; 5 &amp; 4 \\ 4 &amp; 4 &amp; 3 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}, \therefore \text{eigenvalue is } 3</math></p> <p>(b) Either <math>\begin{vmatrix} -8 &amp; 0 &amp; 4 \\ 0 &amp; -4 &amp; 4 \\ 4 &amp; 4 &amp; -6 \end{vmatrix} = -8(24 - 16) + 4(16) = -64 + 64 = 0</math></p> <p>Alt(b) or <math>\begin{vmatrix} 1 - \lambda &amp; 0 &amp; 4 \\ 0 &amp; 5 - \lambda &amp; 4 \\ 4 &amp; 4 &amp; 3 - \lambda \end{vmatrix} = 0 \Rightarrow</math></p> <p><math>(1 - \lambda)(5 - \lambda)(3 - \lambda) - 16(1 - \lambda) - 16(5 - \lambda) = 0</math></p> <p><math>\Rightarrow (3 - \lambda)(\lambda - 9)(\lambda + 3) = 0 \Rightarrow \lambda \text{ is an eigenvalue}</math></p> <p><math>\begin{pmatrix} x \\ y \\ z \end{pmatrix}</math> eigenvector <math>\Rightarrow x + 4z = 9x, 5y + 4z = 9y, 4x + 4y + 3z = 9z</math></p> <p>At least two of these equations</p> <p>Attempt to solve <math>z = 2x, z = y, 2x + 2y = 3z</math></p> <p><math>\therefore \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}</math></p> <p>(c) Make e.vectors unit to obtain <math>\mathbf{P} = \begin{pmatrix} \frac{2}{3} &amp; \frac{1}{3} &amp; \frac{2}{3} \\ \frac{3}{3} &amp; \frac{2}{3} &amp; \frac{1}{3} \\ -\frac{2}{3} &amp; \frac{2}{3} &amp; -\frac{2}{3} \end{pmatrix}</math> columns in any order</p> <p><math>\mathbf{D} = \begin{pmatrix} 3 &amp; 0 &amp; 0 \\ 0 &amp; 9 &amp; 0 \\ 0 &amp; 0 &amp; \lambda_3 \end{pmatrix}</math>, where <math>\lambda_3 = -3</math>, <math>\mathbf{P}</math> and <math>\mathbf{D}</math> consistent</p> | <p>M1A1, A1 (3)</p> <p>M1 A1 (2)</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>A1 (3)</p> <p>M1, A1ft</p> <p>M1, A1, B1 (5)</p> |
| Alt             | <p><math>\mathbf{P} = \begin{pmatrix} 2 &amp; 1 &amp; 2 \\ -2 &amp; 2 &amp; 1 \\ 1 &amp; 2 &amp; -2 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 27 &amp; 0 &amp; 0 \\ 0 &amp; 81 &amp; 0 \\ 0 &amp; 0 &amp; -27 \end{pmatrix}, \mathbf{P} \text{ and } \mathbf{D} \text{ consistent}</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | <p>M1A1ft,<br/>M1A1, B1</p> <p>(13 marks)</p>                                                                             |

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| Question Number | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Marks                                                                                                   |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| 6. (a)          | <p>For <math>n = 1</math> <math>2^5 + 5^2 = 57</math>, which is divisible by 3</p> <p>Assume true for <math>n = k</math> <math>(k + 1)</math>th term is <math>2^{3k+5} + 5^{k+2}</math></p> <p><math>(k + 1)</math>th term <math>\pm</math> <math>k</math>th term <math>= 2^{3k+5} + 5^{k+2} \pm 2^{3k+2} + 5^{k+1}</math></p> $= 2^{3k+2}(2^3 \pm 1) + 5^{k+1}(5 \pm 1)$ <p><math>= 6(2^{3k+2} + 5^{k+1}) + 3 \cdot 2^{3k+2}</math> or <math>= 4(2^{3k+2} + 5^{k+1}) + 3 \cdot 2^{3k+2}</math></p> <p>which is divisible by 3 <math>\Rightarrow (k + 1)</math>th term is divisible by 3</p> <p>Thus by induction true for all <math>n</math> <span style="float: right;">cso</span></p> | <p>M1, A1</p> <p>B1</p> <p>M1</p> <p>M1, A1</p> <p>M1</p> <p>A1</p> <p>B1 <b>(9)</b></p>                |
| (b)             | <p>For <math>n = 1</math> RHS = <math>\begin{pmatrix} -2 &amp; -1 \\ 9 &amp; 4 \end{pmatrix}</math></p> <p>Assume true for <math>n = k</math></p> $\begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix}^{k+1} = \begin{pmatrix} -2 & -1 \\ 9 & 4 \end{pmatrix} \begin{pmatrix} 1-3k & -k \\ 9k & 3k+1 \end{pmatrix} = \begin{pmatrix} -2-3k & 2k-3k-1 \\ 9+9k & -9k+12k+4 \end{pmatrix}$ $= \begin{pmatrix} 1-3(k+1) & -(k+1) \\ 9(k+1) & 3(k+1)+1 \end{pmatrix}$ <p><math>\therefore</math> If true for <math>k</math> then true for <math>k + 1</math> <math>\therefore</math> by induction true for all <math>n</math></p>                                                                  | <p>B1</p> <p>M1 A3/2/1/0<br/>(-1each error)</p> <p>B1</p> <p>B1 <b>(7)</b></p> <p><b>(16 marks)</b></p> |

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| Question Number | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Marks                                                                                                                                                                                                                                                                            |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7.              | <p>(a) <math>\overrightarrow{AB} = 5\mathbf{i} + 3\mathbf{j}</math>    <math>\overrightarrow{AC} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}</math>    or    <math>\overrightarrow{BC} = -2\mathbf{i} - \mathbf{j} - \mathbf{k}</math></p> <p><math>\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} &amp; \mathbf{j} &amp; \mathbf{k} \\ 5 &amp; 3 &amp; 0 \\ 3 &amp; 2 &amp; -1 \end{vmatrix} = -3\mathbf{i} + 5\mathbf{j} + \mathbf{k}</math></p> <p><math>\therefore \mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})</math></p> <p>(b) Volume = <math>\frac{1}{6} \overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})</math>    <math>\overrightarrow{AD} = 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}</math></p> <p><math>= \frac{1}{6} (2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) \cdot (-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})</math></p> <p><math>= \frac{11}{6}</math></p> <p>(c) <math>\mathbf{r} \cdot (-3\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = (2\mathbf{i} + \mathbf{j}) \cdot (-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})</math></p> <p><math>= -1</math></p> <p>(d) <math>[\mathbf{i}(1 - 3\lambda) + \mathbf{j}(2 + 5\lambda) + \mathbf{k}(1 + \lambda)] \cdot (-3\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = -1</math></p> <p><math>-3 + 9\lambda + 10 + 25\lambda + 3 + \lambda = -1</math></p> <p><math>35\lambda + 10 = -1 \quad \Rightarrow \quad \lambda = -\frac{11}{35}</math></p> <p><math>\therefore \text{E is } \left( \frac{68}{35}, \frac{15}{35}, \frac{94}{35} \right)</math></p> <p>(e) Distance = <math>-\frac{11}{35}   -3\mathbf{i} + 5\mathbf{j} + \mathbf{k}   = \frac{11\sqrt{35}}{35}</math>    (Ⓢ)</p> <p>(f) <math>\lambda = 2 \times \left( -\frac{11}{35} \right) = -\frac{22}{35}</math></p> <p><math>\mathbf{r}_{D'} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + -\frac{22}{35} (-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})</math></p> <p><math>D' \text{ is } \left( \frac{101}{35}, -\frac{40}{35}, \frac{83}{35} \right)</math></p> | <p>M1, A1</p> <p>B1 ft    <b>(3)</b></p> <p>B1</p> <p>M1</p> <p>A1    <b>(3)</b></p> <p>M1, A1 ft</p> <p>A1    <b>(3)</b></p> <p>M1, A1 ft</p> <p>M1</p> <p>A1    <b>(4)</b></p> <p>M1 A1    <b>(2)</b></p> <p>B1</p> <p>M1</p> <p>A1    <b>(3)</b></p> <p><b>(18 marks)</b></p> |

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