

Question

Given that

$$\sec^2 x - (1 + \sqrt{3}) \tan x + \sqrt{3} = 1,$$

show that either $\tan x = 1$ or $\tan x = \sqrt{3}$.

proof

Handwritten mathematical proof showing the derivation of $\tan x = 1$ or $\tan x = \sqrt{3}$ from the given equation.

Given: $\sec^2 x - (1 + \sqrt{3}) \tan x + \sqrt{3} = 1$

Step 1: $\Rightarrow (\tan^2 x + 1) - (1 + \sqrt{3}) \tan x + \sqrt{3} = 1$

Step 2: $\Rightarrow \tan^2 x - (1 + \sqrt{3}) \tan x + \sqrt{3} = 0$

Step 3: By the quadratic formula

Step 4: $\Rightarrow \tan x = \frac{-(1 + \sqrt{3}) \pm \sqrt{(1 + \sqrt{3})^2 - 4(1)(\sqrt{3})}}{2 \times 1}$

Step 5: $\Rightarrow \tan x = \frac{1 + \sqrt{3} \pm \sqrt{1 + 2\sqrt{3} + 3 - 4\sqrt{3}}}{2}$

Step 6: $\Rightarrow \tan x = \frac{1 + \sqrt{3} \pm \sqrt{4 - 2\sqrt{3}}}{2}$

Step 7: $\Rightarrow \tan x = \frac{1 + \sqrt{3} \pm \sqrt{(2 - \sqrt{3})^2}}{2}$

Step 8: $\Rightarrow \tan x = \frac{1 + \sqrt{3} \pm (2 - \sqrt{3})}{2}$

Step 9: $\Rightarrow \tan x = \frac{1 + \sqrt{3} + 2 - \sqrt{3}}{2}$ or $\tan x = \frac{1 + \sqrt{3} - 2 + \sqrt{3}}{2}$

Step 10: $\Rightarrow \tan x = \frac{3}{2}$ or $\tan x = \frac{-1}{2}$

Step 11: $\Rightarrow \tan x = 1$ or $\tan x = \sqrt{3}$