

Question

Determine the two real roots of the equation

$$\sqrt{3x^2 - 4x + 34} + \sqrt{3x^2 - 4x - 11} = 9.$$

$$x = 3, -\frac{5}{3}$$

$\sqrt{3x^2 - 4x + 34} + \sqrt{3x^2 - 4x - 11} = 9$ **I**

• CONSIDER

$$\Rightarrow (3x^2 - 4x + 34) - (3x^2 - 4x - 11) \equiv 45 \quad (\text{Note: } 4x \text{ and } -4x)$$

$$\Rightarrow [\sqrt{3x^2 - 4x + 34}]^2 - [\sqrt{3x^2 - 4x - 11}]^2 \equiv 45$$

• DIFFERENCE OF SQUARES

$$\Rightarrow [\sqrt{3x^2 - 4x + 34} - \sqrt{3x^2 - 4x - 11}] [\sqrt{3x^2 - 4x + 34} + \sqrt{3x^2 - 4x - 11}] \equiv 45$$

• USE THE EQUATION WE ARE TRYING TO SOLVE IN THE ABOVE IDENTIFY

WILL TURN IT INTO AN EQUATION

$$\Rightarrow 9 [\sqrt{3x^2 - 4x + 34} - \sqrt{3x^2 - 4x - 11}] = 45$$

$$\Rightarrow \sqrt{3x^2 - 4x + 34} - \sqrt{3x^2 - 4x - 11} = 5$$
 II

Adding (I) & (II) YIELDS

$$\Rightarrow 2\sqrt{3x^2 - 4x + 34} = 14$$

$$\Rightarrow \sqrt{3x^2 - 4x + 34} = 7$$

$$\Rightarrow 3x^2 - 4x + 34 = 49$$

$$\Rightarrow 3x^2 - 4x - 15 = 0$$

$$\Rightarrow (3x + 5)(x - 3) = 0$$

$x = 3$ or $x = -\frac{5}{3}$

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$\sqrt{3x^2 - 4x + 34} + \sqrt{3x^2 - 4x - 11} = 9$

$$\Rightarrow \sqrt{(3x^2 - 4x - 11) + 45} + \sqrt{3x^2 - 4x - 11} = 9$$

Let $u^2 = 3x^2 - 4x - 11$

$$\Rightarrow \sqrt{u^2 + 45} + u = 9$$

$$\Rightarrow \sqrt{u^2 + 45} = 9 - u$$

$$\Rightarrow u^2 + 45 = (9 - u)^2 \quad (u \text{ is REAL})$$

$$\Rightarrow u^2 + 45 = 81 - 18u + u^2$$

$$\Rightarrow 18u = 36$$

$$\Rightarrow u = 2$$

$$\Rightarrow u^2 = 4$$

$$\Rightarrow 3x^2 - 4x - 11 = 4$$

$$\Rightarrow 3x^2 - 4x - 15 = 0$$

$$\Rightarrow (3x + 5)(x - 3) = 0$$

$x = 3$ or $x = -\frac{5}{3}$

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