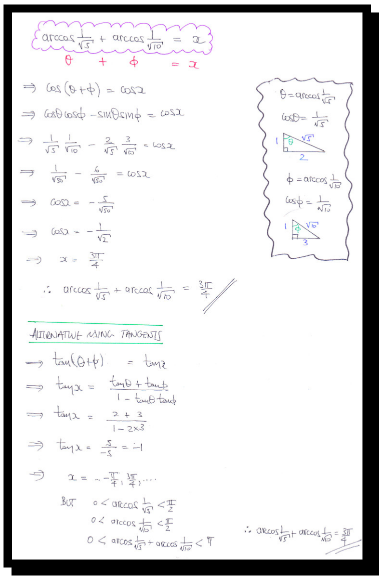
Question (*+)**

Use a detailed method to show that

$$\arccos\left(5^{-\frac{1}{2}}\right) + \arccos\left(10^{-\frac{1}{2}}\right) = \frac{3\pi}{4}.$$

$\boxed{\text{proof}}$



$\arccos\frac{1}{\sqrt{5}} + \arccos\frac{1}{\sqrt{10}} = x$

$\theta + \phi = x$

$\Rightarrow \cos(\theta+\phi) = \cos x$

$\Rightarrow \cos\theta\cos\phi - \sin\theta\sin\phi = \cos x$

$\Rightarrow \frac{1}{\sqrt{5}}\frac{1}{\sqrt{10}} - \frac{2}{\sqrt{5}}\frac{3}{\sqrt{10}} = \cos x$

$\Rightarrow \frac{1}{\sqrt{50}} - \frac{6}{\sqrt{50}} = \cos x$

$\Rightarrow \cos x = -\frac{5}{\sqrt{50}}$

$\Rightarrow \cos x = -\frac{1}{\sqrt{2}}$

$\Rightarrow x = \frac{3\pi}{4}$

$\therefore \arccos\frac{1}{\sqrt{5}} + \arccos\frac{1}{\sqrt{10}} = \frac{3\pi}{4}$

$\theta = \arccos\frac{1}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

$\phi = \arccos\frac{1}{\sqrt{10}}$

$\cos\phi = \frac{1}{\sqrt{10}}$

ALTERNATIVE USING TANGENTS

$\Rightarrow \tan(\theta+\phi) = \tan x$

$\Rightarrow \tan x = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

$\Rightarrow \tan x = \frac{2+3}{1-2\times 3}$

$\Rightarrow \tan x = \frac{5}{-5} = -1$

$\Rightarrow x = \dots -\frac{\pi}{4}, \frac{3\pi}{4}, \dots$

BUT $0 < \arccos\frac{1}{\sqrt{5}} < \frac{\pi}{2}$

$0 < \arccos\frac{1}{\sqrt{10}} < \frac{\pi}{2}$

$0 < \arccos\frac{1}{\sqrt{5}} + \arccos\frac{1}{\sqrt{10}} < \pi$

$\therefore \arccos\frac{1}{\sqrt{5}} + \arccos\frac{1}{\sqrt{10}} = \frac{3\pi}{4}$