

Question

By showing a detailed method, sum the following series.

$$\frac{2}{1} + \frac{3}{2} + \frac{4}{4} + \frac{5}{8} + \frac{6}{16} + \frac{7}{32} \dots$$

6

Handwritten solution for the sum of the series $\frac{2}{1} + \frac{3}{2} + \frac{4}{4} + \frac{5}{8} + \frac{6}{16} + \frac{7}{32} + \dots$.

Let $S = \frac{2}{1} + \frac{3}{2} + \frac{4}{4} + \frac{5}{8} + \frac{6}{16} + \frac{7}{32} + \dots$

$-\frac{1}{2}S = -\frac{2}{2} - \frac{3}{4} - \frac{4}{8} - \frac{5}{16} - \frac{6}{32} - \dots$

Then $S - \frac{1}{2}S = 2 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$

$\Rightarrow \frac{1}{2}S = 2 + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \right)$

This is a G.P. with $a = \frac{1}{2}$ and $r = \frac{1}{2}$

$S_{\infty} = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

$\Rightarrow \frac{1}{2}S = 2 + 1$

$\Rightarrow \frac{1}{2}S = 3$

$\Rightarrow S = 6$