Question

By showing a detailed method, sum the following series.

$$\frac{2}{1}+\frac{3}{2}+\frac{4}{4}+\frac{5}{8}+\frac{6}{16}+\frac{7}{32}\ ...$$

$\boxed{6}$

LET $S = \frac{2}{1} + \frac{3}{2} + \frac{4}{4} + \frac{5}{8} + \frac{6}{16} + \frac{7}{32} + \cdots$

$-\frac{1}{2}S = \quad -\frac{2}{2} - \frac{3}{4} - \frac{4}{8} - \frac{5}{16} - \frac{6}{32} - \cdots$

THUS $S - \frac{1}{2}S = 2 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$

$\Rightarrow \frac{1}{2}S = 2 + \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots\right)$

This is a G.P. WITH $a = \frac{1}{2}$
$r = \frac{1}{2}$

$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

$\Rightarrow \frac{1}{2}S = 2 + 1$

$\Rightarrow \frac{1}{2}S = 3$

$\Rightarrow S = 6$