

Question 7

The product operator \prod , is defined as

$$\prod_{i=1}^k [u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \dots \times u_{k-1} \times u_k.$$

Evaluate, showing a clear method

$$\prod_{r=2}^{\infty} \left[1 - \frac{2}{r(r+1)} \right].$$

$$\boxed{\frac{1}{3}}$$

Handwritten solution for the infinite product problem:

$$\begin{aligned} \prod_{r=2}^{\infty} \left[1 - \frac{2}{r(r+1)} \right] &= \lim_{k \rightarrow \infty} \left[\prod_{r=2}^k \left[1 - \frac{2}{r(r+1)} \right] \right] = \lim_{k \rightarrow \infty} \left[\prod_{r=2}^k \frac{(r-1)(r+2)}{r(r+1)} \right] \\ &= \lim_{k \rightarrow \infty} \left[\frac{1 \times 4}{2 \times 3} \times \frac{2 \times 5}{3 \times 4} \times \frac{3 \times 6}{4 \times 5} \times \frac{4 \times 7}{5 \times 6} \times \dots \times \frac{(k-1)(k+2)}{k(k+1)} \right] \\ &= \lim_{k \rightarrow \infty} \left[\frac{1}{2} \times \frac{4}{3} \times \frac{3}{4} \times \frac{5}{5} \times \frac{6}{6} \times \dots \times \frac{k}{k} \times \frac{(k+2)}{(k+1)} \right] \\ &= \lim_{k \rightarrow \infty} \left[\frac{1 \times 2}{2 \times 3} \times \frac{(k+1)(k+2)}{2 \times 3} \right] = \lim_{k \rightarrow \infty} \left(\frac{k+2}{3k} \right) \\ &= \frac{1}{3} \end{aligned}$$