Question

By showing a detailed method, sum the following series.

$$\frac{\pi^2}{2^2\,2!} - \frac{\pi^4}{2^4\,4!} + \frac{\pi^6}{2^6\,6!} + \frac{\pi^8}{2^8\,8!} + \ldots + \frac{(-1)^{n+1}\pi^{2n}}{2^{2n}(2n)!} + \ldots$$

$\boxed{1}$

$$\boxed{S = \frac{\pi^2}{2!\,2^2} - \frac{\pi^4}{4!\,2^4} + \frac{\pi^6}{6!\,2^6} - \frac{\pi^8}{8!\,2^8} + \ldots + \frac{\pi^{2n}(-1)^{n+1}}{(2n)!\,2^{2n}} + \ldots}$$

- CONSIDER THE EXPANSION OF $\cos x$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots$$

$$\frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \frac{x^8}{8} + \ldots = 1 - \cos x$$

- LET $x = \frac{\pi}{2}$

$$\frac{\left(\frac{\pi}{2}\right)^2}{2!} - \frac{\left(\frac{\pi}{2}\right)^4}{4!} + \frac{\left(\frac{\pi}{2}\right)^6}{6!} + \frac{\left(\frac{\pi}{2}\right)^8}{8!} + \ldots = 1 - \cancel{\cos\frac{\pi}{2}}$$

$$\therefore \frac{\pi^2}{2^2 2!} - \frac{\pi^4}{2^4 4!} + \frac{\pi^6}{2^6 6!} - \frac{\pi^8}{2^8 8!} + \ldots = 1$$