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Question

By showing a detailed method, sum the following series.

$$\sum_{r=1}^{\infty} \left[\frac{2^r}{(r+1)!} \right].$$

$$\frac{1}{2}(e^2-3)$$

$$\begin{split} \zeta &= \sum_{t=1}^{\infty} \left(\frac{t^{t}}{6t!}\right) = \frac{2}{2^{t}} + \frac{u}{3!} + \frac{8}{4!} + \frac{16}{5!} + \frac{32}{6!} \\ \implies \zeta^{5} &= \frac{2^{t}}{2!} + \frac{2^{3}}{3!} + \frac{2^{3}}{4!} + \frac{2^{u}}{5!} + \frac{2^{s}}{6!} + \dots \\ \implies 2^{5} &= \frac{2^{2}}{2!} + \frac{2^{3}}{3!} + \frac{2^{3}}{4!} + \frac{2^{u}}{5!} + \frac{2^{s}}{6!} + \dots \\ \implies 2^{5} &= \frac{2^{2}}{2!} + \frac{2^{3}}{3!} + \frac{2^{3}}{4!} + \frac{2^{5}}{5!} + \frac{2^{5}}{6!} + \dots \\ \implies 2^{5} &= 1 + \frac{2^{t}}{1!} = 1 + \frac{2^{1}}{1!} + \frac{2^{3}}{2!} + \frac{2^{3}}{3!} + \frac{2^{4}}{4!} + \frac{2^{5}}{5!} + \frac{2^{5}}{6!} + \dots \\ \implies 2^{5} &= 8 + 3 = 8 \end{split}$$

$$\implies \zeta^{5} &= \frac{8^{2} - 3}{2!}$$