Question

By showing a detailed method, sum the following series.

$$\sum_{r=1}^{\infty}\left[\frac{2^r}{(r+1)!}\right].$$

$$\boxed{\frac{1}{2}\left(e^2-3\right)}$$

$$\boxed{S = \sum_{r=1}^{\infty}\frac{2^r}{(r+1)!} = \frac{2}{2!}+\frac{4}{3!}+\frac{8}{4!}+\frac{16}{5!}+\frac{32}{6!}}$$

$$\Rightarrow S = \frac{2^1}{2!}+\frac{2^2}{3!}+\frac{2^3}{4!}+\frac{2^4}{5!}+\frac{2^5}{6!}+\dots$$

$$\Rightarrow 2S = \frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}+\frac{2^5}{5!}+\frac{2^6}{6!}+\dots$$

$$\Rightarrow 2S+1+\frac{2^1}{1!} = 1+\frac{2^1}{1!}+\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}+\frac{2^5}{5!}+\frac{2^6}{6!}+\dots$$

$$\Rightarrow 2S+3 = e^2$$

$$\Rightarrow S = \frac{e^2-3}{2}$$