

Question

The straight lines L_1 and L_2 have respective equations

$$4x + 2y = a \quad \text{and} \quad 5x + 4y = b.$$

It is given that L_1 and L_2 meet at the point P .

Express a in terms of b , given further that P lies in the second quadrant and is equidistant from the coordinate axes.

$$a = 2b$$

Handwritten solution showing the steps to solve the system of equations and find the relationship between a and b .

$$\begin{aligned} 4x + 2y &= a \\ 5x + 4y &= b \end{aligned} \Rightarrow \begin{aligned} 8x + 4y &= 2a \\ 5x + 4y &= b \end{aligned} \Rightarrow 3x = 2a - b \Rightarrow x = \frac{2}{3}a - \frac{1}{3}b$$

↓

$$\begin{aligned} 20x + 10y &= 5a \\ 20x + 16y &= 4b \end{aligned} \Rightarrow \begin{aligned} 6y &= 4b - 5a \\ y &= \frac{2}{3}b - \frac{5}{6}a \end{aligned}$$

IN THE SECOND QUADRANT & EQUIDISTANT FROM BOTH AXES
IMPLIES THE INTERCEPT IS ON THE LINE $y = -x$

THUS $\frac{2}{3}b - \frac{5}{6}a = -\left(\frac{2}{3}a - \frac{1}{3}b\right)$

$$\begin{aligned} \frac{2}{3}b - \frac{5}{6}a &= -\frac{2}{3}a + \frac{1}{3}b \\ \frac{1}{3}b &= \frac{1}{6}a \\ a &= 2b \end{aligned}$$