Question

Find in exact simplified form an exact expression for the sum of the first n terms of the following series

$$1 + 11 + 111 + 1111 + 11111 + \ldots$$

$$\boxed{S_n = \frac{1}{81}\left[10^{n+1} - 10 - 9n\right]}$$

$$S_n = 1 + 11 + 111 + 1111 + 11111 + \ldots$$

$$\Rightarrow S_n = \left(\tfrac{1}{9}\times 9\right) + \left(\tfrac{1}{9}\times 99\right) + \left(\tfrac{1}{9}\times 999\right) + \ldots + \left(\tfrac{1}{9}\times \underbrace{999\ldots999}_{\text{(n DIGITS)}}\right)$$

$$\Rightarrow S_n = \frac{1}{9}\left[9 + 99 + 999 + \ldots + \underbrace{999\ldots999}_{\text{n DIGITS}}\right]$$

$$\Rightarrow S_n = \frac{1}{9}\left[(10^1-1) + (10^2-1) + (10^3-1) + \ldots (10^n-1)\right]$$

$$\Rightarrow S_n = \frac{1}{9}\left[\underbrace{[10^1+10^2+10^3+\ldots+10^n]}_{\text{G.P } a=10,\ r=10,\ S_n=\frac{a(r^n-1)}{r-1}} - [\underbrace{1+1+\ldots+1}_{\text{n TERMS}}]\right]$$

$$\Rightarrow S_n = \frac{1}{9}\left[\frac{10(10^n-1)}{10-1} - n\right]$$

$$\Rightarrow S_n = \frac{1}{9}\left[\frac{10^{n+1}-10}{9} - n\right]$$

$$\Rightarrow S_n = \frac{1}{81}\left(10^{n+1} - 10 - 9n\right)$$

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