Question non calculator

By using trigonometric identities, show that

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\, dx = \frac{1}{8}(16 - 3\pi).$$

$\boxed{\text{proof}}$

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\, dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\sin^6 x}{\sin^2 x \cos^2 x} + \frac{\cos^6 x}{\sin^2 x \cos^2 x}\, dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{\sin^4 x}{\cos^2 x} + \frac{\cos^4 x}{\sin^2 x}\, dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{(1-\cos^2 x)^2}{\cos^2 x} + \frac{(1-\sin^2 x)^2}{\sin^2 x}\, dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{1 - 2\cos^2 x + \cos^4 x}{\cos^2 x} + \frac{1 - 2\sin^2 x + \sin^4 x}{\sin^2 x}\, dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sec^2 x \underline{-2} + \underline{\underline{\cos^2 x}} + \operatorname{cosec}^2 x \underline{-2} + \underline{\underline{\sin^2 x}}\, dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} -3 + \sec^2 x + \operatorname{cosec}^2 x\, dx$$

$$= \left[-3x + \tan x - \cot x\right]_{\pi/8}^{\pi/4} = \left[-3x + \tan x - \frac{1}{\tan x}\right]_{\pi/8}^{\pi/4} =$$

$$= \left[-3x + \frac{\tan^2 x - 1}{\tan x}\right]_{\pi/8}^{\pi/4} = \left[-3x - 2\left(\frac{1 - \tan^2 x}{2\tan x}\right)\right]_{\pi/8}^{\pi/4}$$

$$= \left[-3x - \frac{2}{\tan 2x}\right]_{\pi/8}^{\pi/4} = \left(-\frac{3\pi}{4} - 0\right) - \left(-\frac{3\pi}{8} - 2\right) = 2 - \frac{3\pi}{8}$$

$$= \frac{1}{8}(16 - 3\pi)$$