Question

The product operator $\prod$, is defined as

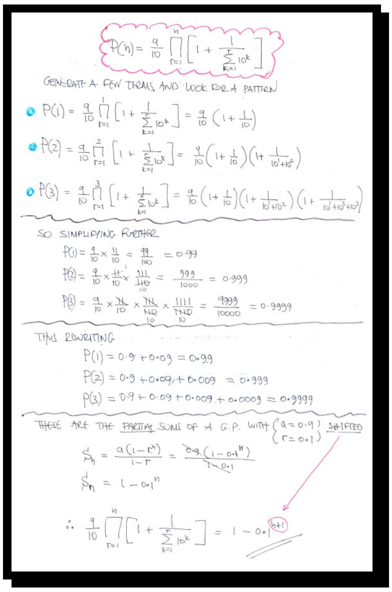
$$\prod_{i=1}^{k}\left[u_i\right] = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k .$$

A sequence of numbers, $P(1)$, $P(2)$, $P(3)$ ... $P(n)$ is defined by the equation

$$P(n) = \frac{9}{10}\prod_{r=1}^{n}\left[1 + \left[\sum_{k=1}^{r} 10^k\right]^{-1}\right].$$

Express $P(n)$ in a simplified form not involving a sigma or product operators.

$$\boxed{P(n) = 1 - 0.1^{n+1}}$$



$$P(n) = \frac{9}{10}\prod_{r=1}^{n}\left[1 + \frac{1}{\sum_{k=1}^{r} 10^k}\right]$$

GENERATE A FEW TERMS AND LOOK FOR A PATTERN

- $P(1) = \frac{9}{10}\prod_{r=1}^{1}\left[1 + \frac{1}{\sum_{k=1}^{r} 10^k}\right] = \frac{9}{10}\left(1 + \frac{1}{10}\right)$
- $P(2) = \frac{9}{10}\prod_{r=1}^{2}\left[1 + \frac{1}{\sum_{k=1}^{r} 10^k}\right] = \frac{9}{10}\left(1 + \frac{1}{10}\right)\left(1 + \frac{1}{10^1+10^2}\right)$
- $P(3) = \frac{9}{10}\prod_{r=1}^{3}\left[1 + \frac{1}{\sum_{k=1}^{r} 10^k}\right] = \frac{9}{10}\left(1 + \frac{1}{10}\right)\left(1 + \frac{1}{10^1+10^2}\right)\left(1 + \frac{1}{10^1+10^2+10^3}\right)$

SO SIMPLIFYING FURTHER

$P(1) = \frac{9}{10} \times \frac{11}{10} = \frac{99}{100} = 0.99$

$P(2) = \frac{9}{10} \times \frac{\cancel{11}}{10} \times \frac{111}{\cancel{110}_{10}} = \frac{999}{1000} = 0.999$

$P(3) = \frac{9}{10} \times \frac{\cancel{11}}{10} \times \frac{\cancel{111}}{\cancel{110}_{10}} \times \frac{1111}{\cancel{1110}_{10}} = \frac{9999}{10000} = 0.9999$

THUS REWRITING

$P(1) = 0.9 + 0.09 = 0.99$

$P(2) = 0.9 + 0.09 + 0.009 = 0.999$

$P(3) = 0.9 + 0.09 + 0.009 + 0.0009 = 0.9999$

THESE ARE THE PARTIAL SUMS OF A G.P. WITH $\begin{pmatrix} a = 0.9 \\ r = 0.1 \end{pmatrix}$ SHIFTED

$S_n = \frac{a(1-r^n)}{1-r} = \frac{\cancel{0.9}(1-0.1^n)}{\cancel{1-0.1}}$

$S_n = 1 - 0.1^n$

$\therefore \frac{9}{10}\prod_{r=1}^{n}\left[1 + \frac{1}{\sum_{k=1}^{r} 10^k}\right] = 1 - 0.1^{n+1}$