## Question

By expressing the integrand in the form $\sec^2 x\, f(\tan x)$, or otherwise, find a simplified expression for

$$\int \frac{3\sin^2 x\cos^2 x}{\left(\cos^3 x+\sin^3 x\right)^2}\,dx\,.$$

$$\boxed{\frac{1}{1-\tan^3 x}+C}$$

$$\int \frac{3\sin^2 x\cos^2 x}{(\cos^3 x+\sin^3 x)^2}\,dx = \int \frac{3\sin^2 x\cos^2 x}{\left[\cos^3 x\left(1-\frac{\sin^3 x}{\cos^3 x}\right)\right]^2}\,dx = \int \frac{3\sin^2 x\cos^2 x}{\cos^6 x(1-\tan^3 x)^2}\,dx$$

$$= \int \frac{\frac{3\sin^2 x\cos^2 x}{\cos^4 x}}{\frac{\cos^6 x}{\cos^4 x}(1-\tan^3 x)^2}\,dx = \int \frac{3\tan^2 x}{\cos^2 x(1-\tan^3 x)^2}\,dx$$

BY SUBSTITUTION OR RECOGNITION

$$= \int 3\sec^2 x\tan^2 x(1-\tan^3 x)^{-2}\,dx = (1-\tan^3 x)^{-1}+C$$

$$= \frac{1}{1-\tan^3 x}+C$$