

Question

$$I = \int_0^1 \left[\prod_{r=1}^{10} (x+r) \right] \left[\sum_{r=1}^{10} \left(\frac{1}{x+r} \right) \right] dx.$$

Show by a detailed method that

$$I = a \times b!,$$

where a and b are positive integers to be found.

The product operator \prod , is defined as

$$\prod_{i=1}^k [u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \dots \times u_{k-1} \times u_k.$$