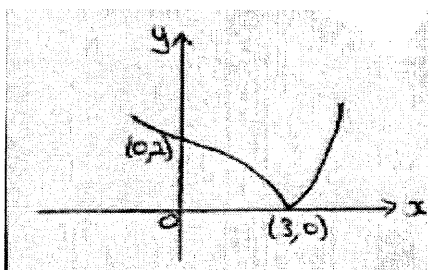
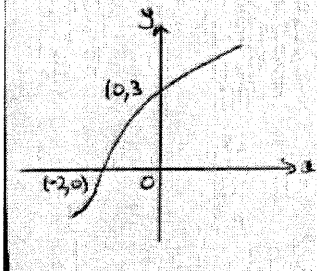
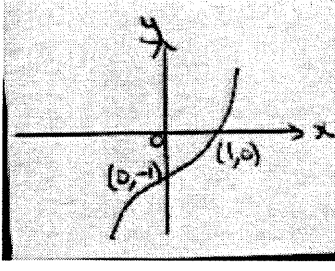


June 2006
6665 Core Mathematics C3
Mark Scheme

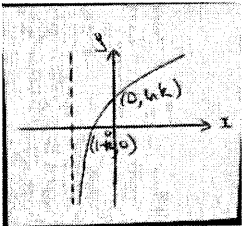
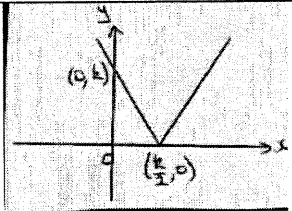
Question Number	Scheme	Marks
1. (a)	$\frac{(3x+2)(x-1)}{(x+1)(x-1)}, = \frac{3x+2}{x+1}$ Notes M1 attempt to factorise numerator, <i>usual rules</i> B1 factorising denominator seen anywhere in (a), A1 given answer If factorisation of denom. not seen, correct answer implies B1	M1B1, A1 (3)
(b)	Expressing over common denominator $\frac{3x+2}{x+1} - \frac{1}{x(x+1)} = \frac{x(3x+2)-1}{x(x+1)}$ [Or "Otherwise" : $\frac{(3x^2 - x - 2)x - (x-1)}{x(x^2 - 1)}$] Multiplying out numerator and attempt to factorise $[3x^2 + 2x - 1 \equiv (3x-1)(x+1)]$ Answer: $\frac{3x-1}{x}$	M1 M1 A1 (3) (6 marks)
2. (a)	$\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}$ Notes B1 $3e^{3x}$ M1 : $\frac{a}{bx}$ A1: $3e^{3x} + \frac{1}{x}$	B1M1A1(3)
(b)	$(5+x^2)^{\frac{1}{2}}$ $\frac{3}{2}(5+x^2)^{\frac{1}{2}} \cdot 2x = 3x(5+x^2)^{\frac{1}{2}} \quad \text{M1 for } kx(5+x^2)^m$	B1 M1 A1 (3) (6 marks)

Question Number	Scheme	Marks
3. (a)	 Mod graph, reflect for $y < 0$ (0, 2), (3, 0) or marked on axes Correct shape, including cusp	M1 A1 A1 (3)
(b)	 Attempt at reflection in $y = x$ Curvature correct (-2, 0), (0, 3) or equiv.	M1 A1 B1 (3)
(c)	 Attempt at 'stretches' (0, -1) or equiv. (1, 0)	M1 B1 B1 (3) (9 marks)
4. (a)	425 °C	B1 (1)
(b)	$300 = 400e^{-0.05t} + 25 \Rightarrow 400e^{-0.05t} = 275$ sub. $T = 300$ and attempt to rearrange to $e^{-0.05t} = a$, where $a \in \mathbb{Q}$ $e^{-0.05t} = \frac{275}{400}$ M1 correct application of logs $t = 7.49$	M1 A1 M1 A1 (4)
(c)	$\frac{dT}{dt} = -20 e^{-0.05t}$ (M1 for $ke^{-0.05t}$) At $t = 50$, rate of decrease = $(\pm) 1.64$ °C/min	M1 A1 A1 (3)
(d)	$T > 25$, (since $e^{-0.05t} \rightarrow 0$ as $t \rightarrow \infty$)	B1 (1) (9 marks)

Ques Num
5.

Question Number	Scheme	Marks
5. (a)	Using product rule: $\frac{dy}{dx} = 2 \tan 2x + 2(2x - 1) \sec^2 2x$ Use of "$\tan 2x = \frac{\sin 2x}{\cos 2x}$" and "$\sec 2x = \frac{1}{\cos 2x}$" $[= 2 \frac{\sin 2x}{\cos 2x} + 2(2x - 1) \frac{1}{\cos^2 2x}]$ Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions $[\Rightarrow 2 \sin 2x \cos 2x + 2(2x - 1) = 0]$ Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. AG	M1 A1 A1 M1 M1 A1* (6)
(b)	$x_1 = 0.2670, x_2 = 0.2809, x_3 = 0.2746, x_4 = 0.2774,$ Note: M1 for first correct application, first A1 for two correct, second A1 for all four correct Max -1 deduction, if ALL correct to > 4 d.p. M1 A0 A1 SC: degree mode: M1 $x_1 = 0.4948$, A1 for $x_2 = 0.4914$, then A0; max 2	M1 A1 A1 (3)
(c)	Choose suitable interval for k: e.g. $[0.2765, 0.2775]$ and evaluate $f(x)$ at these values Show that $4k + \sin 4k - 2$ changes sign and deduction $[f(0.2765) = -0.000087..., f(0.2775) = +0.0057]$ Note: Continued iteration: (no marks in degree mode) Some evidence of further iterations leading to 0.2765 or better M1; Deduction A1	M1 A1 (2) (11 marks)

Question Number	Scheme	Marks
6. (a)	Dividing $\sin^2 \theta + \cos^2 \theta \equiv 1$ by $\sin^2 \theta$ to give $\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} \equiv \frac{1}{\sin^2 \theta}$ Completion: $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta \Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$ AG	M1 A1* (2)
(b)	$\operatorname{cosec}^4 \theta - \cot^4 \theta \equiv (\operatorname{cosec}^2 \theta - \cot^2 \theta)(\operatorname{cosec}^2 \theta + \cot^2 \theta)$ $\equiv (\operatorname{cosec}^2 \theta + \cot^2 \theta)$ using (a) AG Notes: (i) Using LHS = $(1 + \cot^2 \theta)^2 - \cot^4 \theta$, using (a) & elim. $\cot^4 \theta$ M1, conclusion {using (a) again} A1* (ii) Conversion to sines and cosines: needs $\frac{(1 - \cos^2 \theta)(1 + \cos^2 \theta)}{\sin^4 \theta}$ for M1	M1 A1* (2)
(c)	Using (b) to form $\operatorname{cosec}^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta$ Forming quadratic in $\cot \theta$ $\Rightarrow 1 + \cot^2 \theta + \cot^2 \theta \equiv 2 - \cot \theta$ {using (a)} $2 \cot^2 \theta + \cot \theta - 1 = 0$ Solving: $(2 \cot \theta - 1)(\cot \theta + 1) = 0$ to $\cot \theta =$ $\left(\cot \theta = \frac{1}{2} \right)$ or $\cot \theta = -1$ $\theta = 135^\circ$ (or correct value(s) for candidate dep. on 3Ms) Note: Ignore solutions outside range Extra "solutions" in range loses A1✓, but candidate may possibly have more than one "correct" solution.	M1 M1 A1 M1 A1 A1✓ (6) (10 marks)

Question Number	Scheme	Marks
7. (a)	 Log graph: Shape Intersection with -ve x-axis $(0, \ln k), (1 - k, 0)$	B1 dB1 B1
	 Mod graph : V shape, vertex on +ve x-axis $(0, k)$ and $\left(\frac{k}{2}, 0\right)$	B1 B1 (5)
	(b) $f(x) \in \mathbb{R}, -\infty < f(x) < \infty, -\infty < y < \infty$	B1 (1)
	(c) $fg\left(\frac{k}{4}\right) = \ln\left\{k + \left \frac{2k}{4} - k\right \right\} \text{ or } f\left(\left -\frac{k}{2}\right \right)$ $= \ln\left(\frac{3k}{2}\right)$	M1 A1 (2)
	(d) $\frac{dy}{dx} = \frac{1}{x + k}$ Equating (with $x = 3$) to grad. of line; $\frac{1}{3 + k} = \frac{2}{9}$ $k = 1\frac{1}{2}$	B1 M1; A1 A1✓ (4) (12 marks)

Question Number	Scheme	Marks
8. (a)	Method for finding $\sin A$ $\sin A = -\frac{\sqrt{7}}{4}$ Note: First A1 for $\frac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given) Use of $\sin 2A \equiv 2 \sin A \cos A$ $\sin 2A = -\frac{3\sqrt{7}}{8} \text{ or equivalent exact}$ Note: $\pm$ f.t. Requires exact value, dependent on 2nd M	M1 A1 A1 M1 A1√ (5)
(b)(i)	$\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x \cos \frac{\pi}{3} - \sin 2x \sin \frac{\pi}{3} + \cos 2x \cos \frac{\pi}{3} + \sin 2x \sin \frac{\pi}{3}$ $\equiv 2 \cos 2x \cos \frac{\pi}{3}$ [This can be just written down (using factor formulae) for M1A1] $\equiv \cos 2x$ AG Note: M1A1 earned, if $\equiv 2 \cos 2x \cos \frac{\pi}{3}$ just written down, using factor theorem Final A1* requires some working after first result.	M1 A1 A1* (3)
(b)(ii)	$\frac{dy}{dx} = 6 \sin x \cos x - 2 \sin 2x$ or $6 \sin x \cos x - 2 \sin\left(2x + \frac{\pi}{3}\right) - 2 \sin\left(2x - \frac{\pi}{3}\right)$ $= 3 \sin 2x - 2 \sin 2x$ $= \sin 2x \quad \text{AG}$ Note: First B1 for $6 \sin x \cos x$; second B1 for remaining term(s)	B1 B1 M1 A1* (4) (12 marks)

Question Number

1

Beware

So please

Beware awarded

Beware they write

Beware $y - 1 =$

Beware