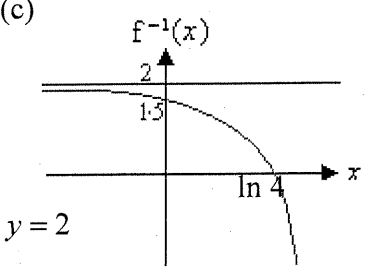


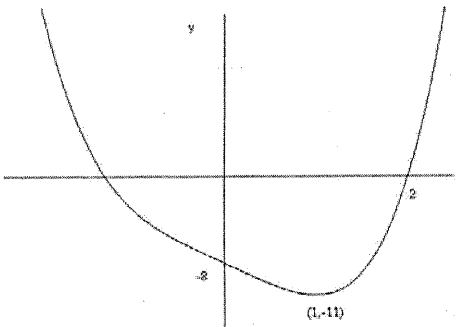
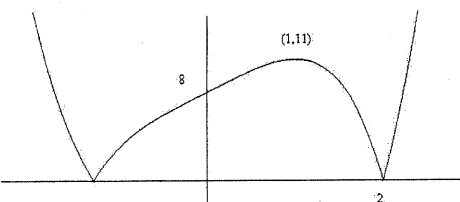
January 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos^2 \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ * (b) $\sin 3\theta = 3 \times \frac{\sqrt{3}}{4} - 4 \left(\frac{\sqrt{3}}{4} \right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent	B1 B1 B1 M1 A1 (5) M1 A1 (2) [7]
2.	(a) $f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ $= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2} = \frac{x^2 + x + 1}{(x+2)^2}$ * (b) $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}, > 0$ for all values of x. (c) $f(x) = \frac{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}$ Numerator is positive from (b) $x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive) Hence $f(x) > 0$	M1 A1, A1 A1 (4) M1 A1, A1 (3) B1 (1) [8]
	Alternative to (b) $\frac{d}{dx}(x^2 + x + 1) = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2 + x + 1 = \frac{3}{4}$ A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2 + x + 1 > 0$ Accept equivalent arguments	M1 A1 A1 (3)

Question Number	Scheme	Marks
3.	(a) $y = \frac{\pi}{4} \Rightarrow x = 2 \sin \frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \Rightarrow P \in C$ Accept equivalent (reversed) arguments. In any method it must be clear that $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ or exact equivalent is used. (b) $\frac{dx}{dy} = 2 \cos y \quad \text{or} \quad 1 = 2 \cos y \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{1}{2 \cos y}$ May be awarded after substitution $y = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{2}} \quad *$ cso (c) $m' = -\sqrt{2}$ $y - \frac{\pi}{4} = -\sqrt{2}(x - \sqrt{2})$ $y = -\sqrt{2}x + 2 + \frac{\pi}{4}$	B1 (1) M1 A1 M1 A1 (4) B1 M1 A1 A1 (4) [9]
4.	(i) $\frac{dy}{dx} = \frac{(9+x^2) - x(2x)}{(9+x^2)^2} \left(= \frac{9-x^2}{(9+x^2)^2} \right)$ $\frac{dy}{dx} = 0 \Rightarrow 9 - x^2 = 0 \Rightarrow x = \pm 3$ $\left(3, \frac{1}{6}\right), \left(-3, -\frac{1}{6}\right)$ Final two A marks depend on second M only (ii) $\frac{dy}{dx} = \frac{3}{2}(1+e^{2x})^{\frac{1}{2}} \times 2e^{2x}$ $x = \frac{1}{2} \ln 3 \Rightarrow \frac{dy}{dx} = \frac{3}{2}(1+e^{\ln 3})^{\frac{1}{2}} \times 2e^{\ln 3} = 3 \times 4^{\frac{1}{2}} \times 3 = 18$	M1 A1 M1 A1 A1, A1 (6) M1 A1 A1 M1 A1 (5) [11]

Question Number	Scheme	Marks
5.	(a) $R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$ $\tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$ accept awrt 1.05 (b) $\sin(x + \text{their } \alpha) = \frac{1}{2}$ $x + \text{their } \alpha = \frac{\pi}{6} \left(\frac{5\pi}{6}, \frac{13\pi}{6} \right)$ $x = \frac{\pi}{2}, \frac{11\pi}{6}$ accept awrt 1.57, 5.76 The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.	M1 A1 M1 A1 (4) M1 A1 M1 A1 (4) [8]

Question Number	Scheme	Marks
6.	(a) $y = \ln(4 - 2x)$ $e^y = 4 - 2x$ leading to $x = 2 - \frac{1}{2}e^y$ Changing subject and removing $\ln$ $y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ * Domain of f^{-1} is (b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in \mathbb{R}$) (c)  (d) $x_1 \approx -0.3704, x_2 \approx -0.3452$ If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion. (e) $x_3 = -0.354\,030\,19 \dots$ $x_4 = -0.350\,926\,88 \dots$ $x_5 = -0.352\,017\,61 \dots$ $x_6 = -0.351\,633\,86 \dots$ $k \approx -0.352$ Alternative to (e) $k \approx -0.352$ Let $g(x) = x + \frac{1}{2}e^x$ $g(-0.3515) \approx +0.0003, g(-0.3525) \approx -0.001$ Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$ $\Rightarrow k = -0.352$ (to 3 dp)	M1 A1 A1 B1 (4) B1 (1) Shape 1.5 $\ln 4$ B1 B1 B1 (4) cao B1, B1 (2) M1 A1 (2) [13] Found in any way M1 A1 (2)

Question Number	Scheme	Marks
7.	(a) $f(-2) = 16 + 8 - 8 (=16) > 0$ $f(-1) = 1 + 4 - 8 (= -3) < 0$ Change of sign (and continuity) $\Rightarrow$ root in interval $(-2, -1)$ ft their calculation as long as there is a sign change (b) $\frac{dy}{dx} = 4x^3 - 4 = 0 \Rightarrow x = 1$ Turning point is $(1, -11)$ (c) $a = 2, b = 4, c = 4$ (d)  (e) 	B1 B1 B1ft (3) M1 A1 A1 (3) B1 B1 B1 (3) Shape ft their turning point in correct quadrant only 2 and -8 B1 (3) Shape B1 (1) [13]

Question Number	Scheme	Marks
8.	(i) $\sec^2 x - \operatorname{cosec}^2 x = (1 + \tan^2 x) - (1 + \cot^2 x)$ $= \tan^2 x - \cot^2 x$ * (ii)(a) $y = \arccos x \Rightarrow x = \cos y$ $x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$ Accept $\arcsin x = \arcsin \cos y$ (b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$	M1 A1 A1 (3) B1 B1 (2) B1 (1) [6]
	<i>Alternatives for (i)</i> Rearranging $\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x$ * cs0 $\left(\text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x} \right)$ $\text{RHS} = \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\cos^2 x \sin^2 x}$ $= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$ $= \text{LHS} *$ or equivalent	M1 A1 A1 (3) M1 A1 A1 (3)