15. P(all shots lie within distance y of centre) $= \left(y^2\right)^n = y^{2n}$ so p.d.f is $\frac{\mathrm{d}}{\mathrm{d}y}y^{2n} = 2ny^{2n-1}$

Hence, Expected area $= \int_0^1 \pi y^2 \times 2ny^{2n-1}\mathrm{d}y = 2n\pi \int_0^1 y^{2n+1}\mathrm{d}y = 2n\pi\left[\frac{y^{2n+2}}{2n+2}\right]_0^1 = \frac{n\pi}{n+1}$

Rejecting the outermost shot, the probability that the remaining $n-1$ shots lie within a circle of radius y is P$(n-2$ shots lie within distance $y) \times$ P(one shot lies within distance y and $y+\delta y) \times$ P(one shot lies further than y from centre) $= n(n-1)y^{2(n-2)} \times (1-y^2) \times 2y\mathrm{d}y$

$= 2n(n-1)\left(y^{2n-3} - y^{2n-1}\right)\mathrm{d}y$

so E(Area) $= 2n(n-1)\pi \int_0^1 \left(y^{2n-1} - y^{2n+1}\right)\mathrm{d}y = 2n(n-1)\pi\left[\frac{y^{2n}}{2n} - \frac{y^{2n+2}}{2n+2}\right]_0^1$

$= 2n(n-1)\pi\left(\frac{1}{2n} - \frac{1}{2n+2}\right) = 2n(n-1)\pi\left(\frac{n+1-n)}{2n(n+1)}\right) = \frac{n-1}{n+1}\pi$

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