7. $y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ so equation becomes $u\frac{dv}{dx} + v\frac{du}{dx} + Puv = Q$

i.e. $\frac{1}{v}\frac{dv}{dx} + \frac{1}{u}\frac{du}{dx} + P = \frac{Q}{uv}$ which becomes $\frac{dv}{dx} = \frac{Q}{u}$ if $\frac{1}{u}\frac{du}{dx} + P = 0$

For $\frac{dy}{dx} - \frac{2y}{x+1} = (x+1)^{3/2}$ we have $P = -\frac{2}{x+1}$, $Q = (x+1)^{3/2}$ (looks like $\frac{5}{2}$ on paper)

so $\frac{1}{u}\frac{du}{dx} - \frac{2}{x+1} = 0 \Rightarrow \int \frac{1}{u}du = \int \frac{2}{x+1}\,dx \Rightarrow \ln u = 2\ln(x+1) \Rightarrow u = (x+1)^2$

so $\frac{dv}{dx} = (x+1)^{-1/2} \Rightarrow v = 2(x+1)^{1/2} + A$ and $y = uv = 2(x+1)^{5/2} + A(x+1)^2$

$y(1) = 0 \Rightarrow 0 = 2^{7/2} + 4A$ so $A = -2^{3/2}$ and $y = 2(x+1)^{5/2} - 2^{3/2}(x+1)^2$

The solution will be valid for $x > -1$

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