

15.

$k$ th head is achieved on  $n$ th toss if previous  $n - 1$  tosses produced  $k - 1$  heads and  $n$ th toss was a

head i.e. Probability =  $\binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \times p = \binom{n-1}{k-1} p^k (1-p)^{n-k}$

Given that it took an even number of tosses to achieve exactly  $k - 1$  heads then the  $k$ th head will occur

on an even numbered toss if we have an odd number of tails followed by a head

so probability =  $p \sum_{r=1}^{\infty} (1-p)^{2r-1} = p \times \frac{(1-p)}{1-(1-p)^2} = \frac{(1-p)}{2-p}$  (sum of infinite G.P)

P(first head occurs on an odd numbered toss) =  $\frac{(1-p)^2}{2-p}$

So P(Exactly two heads are on even numbered tosses) =  $3 \times \left[ \frac{(1-p)}{2-p} \right]^2 \frac{(1-p)^2}{2-p} = \frac{3(1-p)^3}{(2-p)^2}$