

STEP Mathematics Paper 1 1991

9. (i) $x^2 > 2$ and $x^3 > 3 \Rightarrow x^5 > 6$ so statement is false for $n = 5$ hence, $n \leq 4$

(ii) The series $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n^2}$ has $n^2 - n$ terms each of which is $\geq \frac{1}{n^2}$

so the sum $> \frac{n^2 - 1}{n^2} = 1 - \frac{1}{n}$ also $\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n^2} > \frac{1}{n} + 1 - \frac{1}{n} = 1$

$\sum_{n=1}^N \frac{1}{n}$ may be written as $1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \dots + \frac{1}{8}\right) + \dots + \left(\frac{1}{2^{r-2}+1} + \dots + \frac{1}{2^{r-1}}\right)$

giving r brackets each of which has a sum $> \frac{1}{2}$ so taking 19 of these brackets gives a sum > 10

hence, we may take $N = 2^{18}$
