STEP Mathematics Paper 1 1991

10. If T_i is the tension in the portion $A_{i-1}A_i$ and this portion makes an angle θ with the horizontal then, resolving vertically at A_i , by symmetry, $T_i \sin \theta = \frac{1}{2}(2n+3-2i)w$ horizontal component of T_i will remain constant throughout the length of the chain so $T_i \cos \theta = X$ say hence, $\tan \theta = \frac{(2n+3-2i)w}{2X} = \frac{y_{i-1}-y_i}{h} \Rightarrow \frac{(2n+3-2i)h}{y_{i-1}-y_i} = \frac{2X}{w} = K$ (a constant) so taking i=n+1,n,...,1 we have $\frac{h}{y_n-y_{n+1}} = \frac{3h}{y_{n-1}-y_n} = ... = \frac{(2n+1)h}{y_0-y_1}$ as required. Now let A_0 be the origin of coordinates, then $x_1 = h$, $y_1 = -\frac{(2n+1)h}{K}$ $x_2 = 2h$, $y_2 = -\left(\frac{(2n+1)h}{K} + \frac{2nh}{K}\right)$ and in general $x_i = ih$, $y_i = -\sum_{r=1}^{i} \frac{(2n+2-r)h}{K}$ i.e. $y_i = -\frac{h}{K}\left[(2n+2)i - \frac{i(i+1)}{2}\right] = \frac{(i^2-3i-4n)h}{2K} = \frac{\left(\frac{x_i^2}{h^2} - \frac{3x_i}{h} - 4n\right)h}{2K} = \frac{1}{2K}\left(\frac{x_i^2}{h} - 3x_i - \frac{4n}{h}\right)$ i.e. $y = Ax^2 + Bx + C$ where $A = \frac{1}{2Kh}$, $B = -\frac{3}{2K}$ and $C = -\frac{4n}{2Kh}$ so points lie on a parabola.