

11. Let tension in string be T , then by Newton's second law, if acceleration is f ,

$$Mg - T = Mf \text{ and } T - mg = mf \Rightarrow Mg - mg = (m + M)f \Rightarrow f = \left(\frac{m - m}{M + m} \right)g$$

Speed of counterweight is not changed by impact of driver on pile so it has speed v just before driver is jerked into motion again, so if speed just after is V then by conservation of momentum

$$(m + M)v = mv \Rightarrow v = \frac{mv}{m + M}$$

$$\text{Time from release to first impact} = \frac{v}{f} = \frac{(M + m)v}{(M - m)g}$$

$$\text{time between impact and driver moving again} = \frac{2v}{g}$$

$$\text{speed of driver is now } \frac{m^2v}{(m + M)^2} \text{ so time to next impact is } \frac{2m^2v}{(m + M)^2} \times \frac{(M + m)}{(M - m)g} = \frac{2m^2v}{(M + m)(M - m)g}$$

$$\text{total time between first and second impacts} = \frac{2v}{g} + \frac{2mv}{(M - m)g} = T_1 \text{ say}$$

$$\text{and between second and third impacts} = \frac{2mv}{(M + m)g} + \frac{2m^2v}{(M + m)(M - m)g} = \left(\frac{m}{m + M} \right) T_1$$

times between successive impacts thus form a G.P. with common ratio $\frac{m}{m + M}$

$$\text{total time between impacts is thus } \left(\frac{2v}{g} + \frac{2mv}{(M - m)g} \right) \times \frac{1}{1 - \frac{m}{m + M}} = \frac{2v(M + m)}{(M - m)g}$$

$$\text{so time from release to rest} = \frac{(M + m)v}{(M - m)g} + \frac{2v(M + m)}{(M - m)g} = \frac{3v(M + m)}{(M - m)g}$$

i.e. three times the time from release to first impact