

STEP Mathematics Paper 1 1991

13. Considering relative motion of particle and lamina, the force exerted by the lamina on the particle is always perpendicular to its motion so speed of particle relative to lamina will be constant. Hence, if speed of lamina when particle leaves the groove is v then that of the particle will be $v - V$.

So by conservation of momentum $Mv + m(v - V) = mV \Rightarrow (M + m)v = 2mV$
 so speed of lamina is $\frac{2mV}{M+m}$ and that of the particle is $\frac{2mV}{M+m} - V = \frac{(m-M)V}{M+m}$

Consider now when particle is at P .

Reaction on lamina will be $R = \frac{Mv^2}{r} = m\omega^2 r$

Reaction on peg along line of symmetry will be

$$R \sin \theta = m\omega^2 r \sin \theta$$

so total impulse on lamina and peg is

$$\int_0^\pi m\omega^2 r \sin \theta d\theta = [-m\omega^2 r \cos \theta]_0^\pi = 2m\omega^2 r$$

Total time particle is in contact with lamina is $\frac{\pi r}{V}$

so average force is $\frac{2m\omega^2 r V}{\pi r} = \frac{2m\omega^2 V}{\pi}$

