

14. Let $a = 2k$ then we may take $b = k + 1$ with $c = k$, i.e. one triangle
 or taking $b = k + 2$ with $c = k + 1, k$ or $k - 1$ gives 3 triangles, or in general
 taking $b = k + r$ we may take $c = k + (r - 1), k + (r - 2), \dots, k + (1 - r)$ giving
 $k + (r - 1) - (k + (1 - r)) + 1 = 2r - 3$ triangles
 so total number of possible triangles is $1 + 3 + 5 + \dots + 2N - 3$ and $2n - 3 = a - 3$
 Now let $a = 2k + 1$ then we may take

$b = k + r$ with $c = k + (r - 1), k + (r - 2), \dots, k + 2 - r$ giving $2r - 2$ triangles
 so number of possible triangles is $2 + 4 + 6 + \dots = 2N - 2 (= a - 3)$
 so total number of non-degenerate triangles is

$$a = 4; (b, c) = (3, 2)$$

$$a = 5; (b, c) = (4, 3), (4, 2)$$

$$a = 6; (b, c) = (5, 4), (5, 3), (5, 2), (4, 3)$$

$$a = 7; (b, c) = (6, 5), (6, 4), (6, 3), (6, 2), (5, 4), (5, 3)$$

$$\text{i.e. } 1 + 2 + (1 + 3) + 2 + 4 + (1 + 3 + 5) + (2 + 4 + 6) + \dots + (1 + 3 + \dots + 2N - 3) + (2 + 4 + \dots + 2N - 2)$$

$$= (1 + 2) + (1 + 2 + 3 + 4) + (1 + 2 + \dots + 6) = 91 + \dots + 8 + \dots + (1 + \dots + 2N - 2)$$

$$= 3 + 10 = 21 + 36 = \dots = N(2N + 1) = \sum_{r=1}^{N-1} r(2r + 1) = 2 \sum_{r=1}^{N-1} r^2 + \sum_{r=1}^{N-1} r$$

$$= \frac{2}{6}(N - 1)N(2N - 1) + \frac{1}{2}(N - 1)N = \frac{1}{6}N(N - 1)(4N - 2 + 3) = \frac{1}{6}N(N - 1)(4N + 1)$$

so for any value of N , the number of triangles is $\frac{1}{6}N(N - 1)(4N + 1)$

$$N = 3 \text{ gives } \frac{1}{2} \times 2 \times 13 = 13$$

Check!

From above $a = 7$ gives 6 triangles, $a = 6$ gives 4, $a = 5$ gives 2 and $a = 4$ gives 1 i.e. a total of 13

$$\text{Number of possible selections of three rods is } \frac{1}{6}(2N + 1)2N(2N - 1) = \frac{1}{3}N(4N^2 - 1)$$

$$\text{so probability that a triangle can be formed from 3 randomly selected rods is } \frac{\frac{1}{6}N(N - 1)(4N + 1)}{\frac{1}{3}N(4N^2 - 1)}$$

$$\text{i.e. } \frac{(N - 1)(4N + 1)}{2(4N^2 - 1)} \text{ as required.}$$
