15. (i) $S_n > 100$ if number of heads > 5050
Number of heads in 10000 throws has a $B(10000, 0.5)$ distribution which we may approximate by $N(5000, 2500)$ so P(more than 5050 heads) $= P\left(z > \dfrac{50.5}{\sqrt{2500}}\right) = P(z > 1.01) \approx 0.156$

(ii) For n throws, distribution is $B(n, 0.5) \approx N(0.5n, 0.25n)$

P(more than $0.01n$) $= P\left(z > \dfrac{0.005n}{0.5\sqrt{n}}\right) < 0.01$ if $\dfrac{0.005n}{0.5\sqrt{n}}.1.28$

$\Rightarrow 0.005\sqrt{n} > 0.64 \Rightarrow n.\dfrac{0.64^2}{0.005^2} \approx 16400$

Score on *kth* throw is 2 points if $X_k = X_1 =$ a head, -2 points if $X_k = XC_1 =$ a tail and 0 if $X_k \neq X_1$

If X_1 is a head then $X_k = X_1 \Rightarrow X_1 + X_k = 2$

If X_1 is a tail then $X_k = X_1 \Rightarrow X_1 + X_k = -2$

If X_1 and X_k differ then $X_1 + X_k = 0$ so in every case $Y_k = X_1 + X_k$

$P(Y_k = -2) = P(Y_k = 2) = \frac{1}{4}$ and $P(Y_k = 0) = \frac{1}{2}$

so mean of $Y_k = -2 \times \frac{1}{4} + 2\frac{1}{4} = 0$ and variance $= 4\frac{1}{4} + 4\frac{1}{4} - 0^2 = 2$

hence, $Y_2 + Y_3 + \ldots + Y_n$ will have mean 0 and variance $2(n-1)$

so $\sum\limits_{r=2}^{n} Y_r$ has approximate distribution $N(0, 2(n-1))$

$P\left(\sum\limits_{r=2}^{n} Y_r > 0.01(n-1)\right) = P\left(z > \dfrac{0.01(n-1)}{\sqrt{2(n-1)}}\right) = P\left(z > 0.01\sqrt{\dfrac{n-1}{2}}\right)$ which does tend to zero as $n \to \infty$