

16. Since the toad only attempts to cross when it is safe, she is certain to cross without mishap.

$$\begin{aligned} P(\text{frog crosses safely}) &= 0.1 + 0.9 \times 0.1 + 0.9^2 \times 0.2 + 0.9^2 \times \frac{2}{3} \times 0.1 + 0.9^3 \times \left(\frac{2}{3}\right)^2 \times 0.2 + \\ &\quad + 0.9^3 \times \frac{2}{3} \times \frac{1}{3} \times 0.1 + 0.9^4 \times \frac{2}{3} \times \frac{1}{3} \times 0.2 = 0.4081 \end{aligned}$$

$P(\text{both cross safely with frog before the toad}) = P(\text{frog crosses safely before it is safe to do so})$

$$\text{i.e. } 0.9^2 \times \frac{1}{3} \times 0.2 + 0.9^3 \times \left(\frac{2}{3}\right)^2 \times 0.2 + 0.9^4 \times \frac{2}{3} \times \frac{1}{3} \times 0.2 = 0.148$$

$P(\text{frog is run over given he has not arrived after 2 minutes})$

$$\begin{aligned} &= \frac{P(\text{he is run over and has not arrived after 2 minutes})}{P(\text{he has not arrived after 2 minutes})} = \frac{P(\text{he is run over})}{P(\text{not arrived after 2 minutes})} \\ &= \frac{P(\text{he is run over})}{1 - P(\text{he has crossed in first 2 minutes})} = \frac{1 - 0.4081}{1 - \left(0.1 + 0.9 \times 0.1 + 0.9^2 \times \frac{1}{3} \times 0.2 + 0.9^3 \times \left(\frac{2}{3}\right)^2 \times 0.2\right)} \\ &= \frac{0.5919}{0.6048} = 0.9787 \end{aligned}$$
